Sector-Level Accounting Betas and Cost of Equity
in Data-Constrained Emerging Markets:
A Tiered Framework for Colombia

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ABSTRACT

Valuation practice in Colombia requires sector-level cost-of-equity estimates for non-listed firms in a data-constrained environment: limited annual accounting frequency, short time series, sector heterogeneity, and absence of market betas for most firms. Damodaran's emerging-market betas provide international comparability but are not locally calibrated; relying on the international benchmark alone leaves the professional without locally informed sectoral discrimination. We propose CAB-Ke, a tiered accounting-beta framework producing professionally defensible cost-of-equity estimates for the 27 non-financial sectors of the Colombian SIREM panel 2006–2024 (3,704 firms; 44,246 firm-year observations), with explicit reliability classification. The framework integrates Hill-Stone profitability betas, distress-augmented betas, Beaver-Kettler-Scholes accounting variables, GLS combination, and empirical-Bayes shrinkage toward a CAPM-neutral prior. Each sector is assigned to one of five tiers: Tier 1 Strong Professional Evidence (3 sectors), Tier 2 Professional Use (6 sectors), Tier 3 Indicative (2 sectors), Tier 4 Fallback (16 sectors), and Tier 5 Special Treatment (2 financial sectors excluded). Under the strict symmetric dual-factor academic criterion only one sector qualifies; we report this transparently. Cost-of-equity conversion uses Hamada relevering with the Colombian corporate standard rate, Pereiro Adjusted Hybrid CAPM with country-risk premium documented across three sources, and Fisher conversion in USD and COP. Damodaran is preserved as an external comparability benchmark. The framework's contribution is not statistical certainty for every sector but transparent professional estimation under uncertainty, with tier-specific disclosure requirements appropriate for Colombian investment-banking, corporate-valuation, family-office, fairness-opinion, and arbitration practice.

JEL Classification: G12, G15, G31, G32, G33, M41, C52, C53.

Keywords: accounting beta; cost of equity; emerging markets; valuation practice; Adjusted Hybrid CAPM; tiered reliability classification; Bayesian shrinkage; open-interval Vasicek; Colombia.

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I. INTRODUCTION

THE COST OF EQUITY for non-listed firms in emerging markets is one of the most consequential and least empirically validated calculations in corporate finance. Investment bankers, valuation professionals, family offices, transfer-pricing specialists, and tax authorities in Colombia routinely produce sector-level cost-of-equity estimates as inputs to discounted-cash-flow valuations, capital-budgeting decisions, fairness opinions, and arbitration proceedings. The professional standard requires these estimates to be defensible: under the scrutiny of an arbitration panel, a regulatory authority, or a transaction committee, the valuator must be able to explain how the beta was estimated, what its empirical basis is, and what reliability the analyst attributes to it.

Existing practice in Colombia rests on two unsatisfactory alternatives. The first is to import Damodaran's emerging-market sector unlevered betas without local validation. This provides international comparability but is not locally calibrated and does not come accompanied by standard errors, statistical significance tests, or out-of-sample validation against Colombian macroeconomic outcomes. The second is to compute idiosyncratic firm- or sector-level betas without a coherent reliability framework, leaving the professional to defend ad-hoc methodological choices in front of skeptical audiences. Neither option supports the disciplined, transparent disclosure that contemporary professional valuation practice demands.

This paper proposes the Colombian Accounting-Beta Framework for the Cost of Equity, abbreviated CAB-Ke, a tiered hierarchical procedure for estimating sector-level cost of equity using accounting betas on the Sistema de Información y Reporte Empresarial (SIREM) panel 2006–2024. The framework's central premise differs from the standard academic objective. In a developed-market setting with extensive time-series data, transparency of listed firms, and traded market benchmarks, the academic objective is strict statistical validation under conventional thresholds. In an emerging-market setting with annual accounting frequency, short panels, and the absence of market betas for most firms, this strict standard cannot be met for most sectors regardless of methodological care. The professional valuation question, however, still arises.

The framework's central thesis is operational rather than methodological. In data-constrained emerging markets, the appropriate professional standard is not mechanical statistical purity but transparent estimation under uncertainty with explicit reliability classification at each sector. CAB-Ke does not claim to validate every Colombian sector beta. It provides a complete, stratified, transparent sectoral cost-of-equity matrix that distinguishes among three evidence standards—strict academic, strong professional, and emerging-market indicative—and assigns each sector to one of five reliability tiers. This classification serves the valuation profession better than either refusing to estimate when statistical thresholds are not met or delivering point estimates without disclosure of their empirical fragility.

The framework's primary outputs are as follows. For each of the 27 non-financial sectors, we report a final unlevered beta from the tiered procedure, a Hamada-levered beta at the Colombian corporate standard tax rate as headline, a US-dollar cost of equity under Pereiro Adjusted Hybrid CAPM, a Colombian-peso cost of equity under Fisher conversion with long-run inflation expectations, 90% and 95% confidence intervals, the Damodaran benchmark for international comparability, the percentage gap relative to that benchmark, a tier-specific

recommended category, required disclosure language, and—for Tier 4 sectors—an explicit Fallback Reason explaining why local evidence is insufficient.

The paper proceeds as follows. Section II reviews the relevant literature. Section III describes the data and exclusion of financial sectors. Section IV reviews component accounting-beta methodologies. Section V documents the validation arsenal, explicitly distinguishing between academic and professional evidence standards. Section VI introduces the five-tier reliability classification. Section VII develops the five-layer CAB-Ke estimation procedure. Section VIII reports empirical results. Section IX documents the full sectoral cost-of-equity matrix. Section X reports robustness and sensitivity analysis. Section XI documents the professional decision rule for Colombian valuation practice, with tier-specific disclosure templates. Section XII acknowledges framework limitations. Section XIII concludes.

II. LITERATURE REVIEW

A. Accounting Beta Methodologies

The accounting-beta literature originates with Beaver, Kettler, and Scholes (1970), who demonstrated that accounting variables explain a substantial fraction of cross-sectional variation in market betas for U.S.-listed firms. Hill and Stone (1980) formalized the time-series accounting beta as the slope coefficient in a regression of firm-level changes in return on assets on aggregate market changes. Mandelker and Rhee (1984) decomposed the levered accounting beta into operating-leverage, financial-leverage, and intrinsic business-risk components. Cohen, Polk, and Vuolteenaho (2009) showed that accounting cash-flow-based betas explain the cross-section of average returns over long horizons. Ellahie (2021) documents the consistency of earnings-based betas in U.S. data over 1981–2017. For non-listed firms in emerging markets, the accounting beta has been the default methodological recourse. The Colombian context specifically has been studied by Sarmiento-Sabogal and Sadeghi (2014, 2015) and by Sarmiento, Sadeghi, Sandoval, and Cayon (2021), who document unlevered accounting betas and develop cost-of-equity estimates for non-listed firms. Ball, Sadka, and Tseng (2022) argue that macroeconomic indicators rather than aggregate accounting profitability should serve as the market benchmark in segmented or emerging-market settings.

B. Cost of Equity for Emerging Markets

Pereiro (2001, 2006) develops the Adjusted Hybrid CAPM that combines a US equity risk premium with a country risk premium for emerging-market application. Damodaran (2024, 2026) publishes annual emerging-market sector beta estimates along with country risk premium estimates and is the dominant practitioner reference for international comparability across transactions. Bekaert, Hodrick, and Zhang (2009) provide evidence that parsimonious risk-based factor models combining local and global components capture international stock return covariance better than pure global-market approaches. Bai and Green (2020) document that country and industry factors are independent sources of stock return variation in partially integrated emerging markets, with implications for the choice between local and global asset-pricing model specifications. The present paper builds on these foundations: rather than presenting CAB-Ke as a replacement for Damodaran, we present it as a complementary local framework

that produces locally informed sectoral discrimination while preserving Damodaran as an international comparability benchmark.

C. Combination, Shrinkage, and Multiple Testing

The forecast combination literature begins with Bates and Granger (1969) and Cochran (1937) and has been extensively developed by Stock and Watson (2004) and Timmermann (2006). The shrinkage literature in finance originates with Vasicek (1973), who applied Stein-type estimators to security betas, and with Efron and Morris (1973) for hierarchical empirical-Bayes estimation. The multiple-testing correction via false discovery rate is due to Benjamini and Hochberg (1995); we apply this correction to per-sector p-values when classifying sectors at the Strong Professional Evidence tier.

D. Financial Distress and the Credit Cycle

Campbell, Hilscher, and Szilagyi (2008) demonstrated that distress risk premia exhibit a factor structure separate from standard size and book-to-market factors. Our distress-augmented accounting beta operationalizes the credit-cycle dimension by replacing the aggregate ROA factor with an aggregate default probability constructed from firm-level Z-scores in the tradition of Altman (1968) and Ohlson (1980). Z-score parameters in our specification are calibrated on data partially overlapping with the analysis sample; this is a documented limitation discussed in Section XII.

III. DATA AND SECTOR CONSTRUCTION

A. Sample and Filters

Our sample is drawn from the SIREM database maintained by the Colombian Superintendencia de Sociedades, supplemented with the commercial EMIS database for cross-validation. We retain firm-year observations with total assets above 30 million Colombian pesos, complete reporting of balance-sheet and income-statement items required to compute operating return on assets, debt-to-equity ratio within [0, 3], and a balanced-cohort filter requiring each firm to appear in at least seven of the nineteen years 2006–2024. After firm-level filters the cohort comprises 3,704 firms with 44,246 firm-year observations distributed across 30 sectors under the CIIU revision 4 two-digit classification.

After sector-level filters requiring at least 20 firm-year observations per year and at least 8 valid sector-year change observations, we retain 29 sectors with adequate sample. Two of these sectors are excluded from the framework as Tier 5 Special Treatment: CIIU 64 Financial Services and CIIU 66 Auxiliary Financial Services. The exclusion rationale is that the SIREM accounting D/E captures only the firm's own financial obligations and excludes customer deposits, which constitute the bulk of leverage for financial intermediaries. For these sectors, the Hamada relevering formula in equation (12) below would produce economically meaningless levered betas. The final analysis panel comprises 27 non-financial sectors.

We apply year-by-year winsorization at the 1st and 99th percentiles to firm-level operating ROA and equity-to-assets ratios before aggregation. While Adams, Hayunga, Mansi, Reeb, and Verardi (2019) document that univariate

winsorization may underperform multivariate outlier detection in panel finance applications, we adopt the conventional 1%-99% rule as a preprocessing step for comparability with prior accounting-beta studies, and complement it with M-regression using the Tukey biweight loss in the time-series estimation step (Section IV).

Table 1

Sample Construction and Sectoral Coverage

Summary of the non-financial SIREM Colombia sample 2006-2024 after filters. The 27 non-financial CIIU rev. 4 two-digit sectors constitute the analysis panel; two financial sectors (CIIU 64, 66) are excluded as Tier 5 Special Treatment because SIREM's accounting D/E does not capture customer-deposit leverage.

Sample characteristic	Value
Sample period	2006-2024 (19 annual observations)
Firms in cohort	3,704
Firm-year observations	44,246
Analyzed sectors (non-financial, CIIU 2-digit)	27
Excluded sectors (Tier 5 Special Treatment)	CIIU 64, CIIU 66
Mean firms per sector-year	~98
Median sector D/E (non-financial)	0.91
Asset-weighted mean operating ROA	5.4%
Firm-level filters	Assets > 30M COP; complete reporting; D/E ∈ [0, 3]
Cohort filter	Firm in ≥ 7 of 19 years
Sector-level filters	≥ 20 firm-year obs/year; ≥ 8 sector-year changes
Winsorization	Year-by-year 1%-99% on firm-level ratios

B. Aggregation and Factor Construction

For each sector-year (s, t), we construct the asset-weighted operating return on assets:

$$ROA_{s,t} = \Sigma_{i:\, s(i)=s}\ w_{i,t} \cdot X_{i,t} \,/\, A_{i,t} \qquad (1)$$

where i indexes firms, X_{i,t} is operating income, A_{i,t} is total assets, and w_{i,t} is the asset weight. Aggregate market ROA, ROA_{M,t}, is constructed analogously. Aggregate market default probability, PD_{M,t}, is the asset-weighted average of firm-level default probabilities derived from a panel-validated Z-score:

$$z_{i,t} = -1.2011 - 6.7120 \cdot R1_{i,t} - 3.4259 \cdot R4_{i,t} \qquad (2)$$

where R1 = EBIT/Assets and R4 = Equity/Assets. First differences ΔROA_{s,t}, ΔROA_{M,t}, and ΔPD_{M,t} serve as inputs to time-series accounting-beta regressions in Section IV.

IV. COMPONENT ACCOUNTING-BETA METHODOLOGIES

Table 2

Component Accounting-Beta Methodologies

Summary of the four component methodologies used by CAB-Ke. Each component is estimated independently; the combination layer (Section VII.B) integrates them via inverse-variance and GLS.

Component	Specification	Role in CAB-Ke
Hill-Stone (1980)	Time-series slope of ΔROA_s on ΔROA_M (eq. 3); Tukey biweight M-regression; stationary bootstrap inference.	*Profitability factor. Primary signal of procyclical sensitivity to aggregat profitability.*
Distress-augmented	Slope of ΔROA_s on aggregate default probability changes ΔPD_M (eq. 4); Z-score-based PD construction (eq. 2).	*Credit-cycle factor in the spirit of Campbell, Hilscher, and Szilagyi (200 HS in Layer 2.*
Beaver-Kettler-Scholes	Cross-sectional regression of HS beta on sector D/E, log assets, σ_ROA (eq. 5).	*Accounting-variable benchmark. Used as fallback rule input (weight 0.5 check.*
Damodaran benchmark	Damodaran annual adjusted emerging-market sector betas (2026 update); constant per sector over sample.	*External comparability benchmark. Used as fallback rule input (weight international reference.*

The CAB-Ke framework rests on four documented accounting-beta methodologies: Damodaran's adjusted emerging-market beta (as external benchmark and fallback-rule input), classical Hill-Stone accounting beta on the profitability factor, distress-augmented accounting beta in the Campbell-Hilscher-Szilagyi spirit, and the Beaver-Kettler-Scholes accounting-variable specification. We articulate the theoretical motivation for this four-component architecture in Section IV.A and the empirical specifications in Sections IV.B-IV.E.

A. Theoretical Motivation for the Four-Component Framework

The selection of the four component accounting betas is theoretically motivated by three distinct considerations rooted in modern asset pricing theory and the forecast combination literature. We articulate each motivation explicitly before describing the empirical specifications.

First, the Hill-Stone (1980) and distress-augmented (Campbell, Hilscher, and Szilagyi 2008) specifications operationalize the two systematic dimensions of emerging-market equity risk that are economically distinguishable and theoretically separable. The Hill-Stone beta captures the cash-flow news component of equity returns in the sense of Campbell (1991): sectoral sensitivity to aggregate profitability fluctuations represents systematic exposure to news about expected future cash flows. The distress-augmented beta captures the credit-cycle dimension as an intertemporal state variable in the sense of Merton (1973): sectoral sensitivity to aggregate default probability changes represents systematic exposure to a separate priced risk factor that prices time-varying investment opportunities under partial market segmentation. These two factors are not redundant: in our sample, the cross-sectional correlation between $\beta_{HS,s}$ and $\beta_{DA,s}$ is approximately 0.45, indicating that each component captures roughly 80% unique information (computed as $1-\rho^2$ with $\rho \approx 0.45$) after controlling for the other. The decomposition reflects the empirical regularity that emerging-market firms exhibit substantial heterogeneity in both their procyclicality (cash-flow sensitivity) and their distress sensitivity (intertemporal state variable exposure).

Second, the Beaver-Kettler-Scholes (1970) accounting-variables specification serves a complementary cross-sectional benchmarking function. While $\beta_{HS,s}$ and $\beta_{DA,s}$ exploit within-sector temporal variation, $\beta_{BKS,s}$ exploits cross-sectional variation in sector accounting characteristics (leverage, scale, profitability volatility). This combination of within- and between-sector identification is theoretically motivated by the classical bias-variance tradeoff in panel beta estimation: time-series estimators are unbiased but high-variance when panels are short ($T = 19$ in our sample); cross-sectional estimators are lower-variance but potentially biased toward the cross-sectional mean. The inclusion of BKS reduces estimation variance for sectors with weak time-series evidence

at the cost of some bias toward the cross-sectional average—an economically beneficial tradeoff under the data-constrained conditions characteristic of emerging-market accounting panels.

Third, the inclusion of the Damodaran emerging-market benchmark as a fourth component is motivated by the partial integration literature in international asset pricing (Bekaert and Harvey 1995; Bai and Green 2020). Under partial integration, neither the purely local CAPM nor the purely global CAPM is fully consistent with observed cross-sectional patterns; the theoretically appropriate cost of equity blends local and global benchmarks with weights reflecting the degree of integration. Damodaran's adjusted emerging-market beta, calibrated to global emerging-market sectoral composition, serves as the global anchor in this blend; the local components ($\beta_{HS,s}$, $\beta_{DA,s}$, $\beta_{BKS,s}$) provide the local discrimination relative to the global anchor.

The combination weights are derived from the inverse-variance combination principle of Bates and Granger (1969) and Cochran (1937). Under the assumption of unbiased component estimators with known relative precisions, the variance-minimizing linear combination assigns weights proportional to inverse variances. For the Tier 4 fallback specification, where local time-series evidence is insufficient, we calibrate weights of 0.50 (BKS), 0.30 (sectoral median), and 0.20 (Damodaran) to reflect the relative cross-sectional informativeness of each component: BKS receives the largest weight as it leverages the full cross-section of 27 sectors with multiple accounting variables; the sectoral median provides a robust low-variance benchmark from within-sample evidence; the Damodaran weight contributes a substantive but non-dominant international comparability anchor. Section X.M reports sensitivity to alternative weight specifications.

The Vasicek (1973) shrinkage prior ($\beta_0 = 1.0$, $\sigma_0 = 0.40$) merits separate theoretical justification. The prior mean $\beta_0 = 1.0$ is the CAPM-neutral default: under the CAPM, the value-weighted market portfolio has $\beta = 1.0$ by definition, and in the absence of strong sector-specific information, the natural prior expectation for any sector's beta is the market's beta. The prior standard deviation $\sigma_0 = 0.40$ is calibrated to match the empirical cross-sectional dispersion of unlevered betas across emerging-market sectors: Damodaran's (2026) emerging-market sectoral betas exhibit a cross-sectional standard deviation of approximately 0.35-0.45, consistent with our $\sigma_0 = 0.40$ choice. This calibration implies that, under the prior alone, approximately 68% of true sector betas fall in [0.60, 1.40] and approximately 95% fall in [0.20, 1.80], an interval that comfortably contains the full range of Damodaran's emerging-market sectoral betas. Section X.L reports sensitivity to alternative prior specifications, including an Empirical Bayes alternative that derives β_0 and σ_0 from the cross-section of our own data.

B. Hill-Stone Profitability Beta

For each sector s with at least eight valid sector-year change observations, the Hill-Stone beta is the slope coefficient in:

$$\Delta ROA_{s,t} = \alpha_s + \beta_{HS,s} \cdot \Delta ROA_{M,t} + \varepsilon_{s,t} \quad (3)$$

Estimation is by M-regression with the Tukey biweight loss function (tuning constant c = 4.685). Inference for the slope coefficient is by stationary bootstrap with block length L = 4 and 1,000 bootstrap replicates, following Politis and Romano (1994).

C. Distress-Augmented Beta

Following Campbell, Hilscher, and Szilagyi (2008), the distress-augmented accounting beta replaces the aggregate ROA factor with aggregate market default probability:

$$\Delta ROA_{s,t} = \alpha_s + \beta_{DA,s} \cdot \Delta PD_{M,t} + \varepsilon_{s,t} \quad (4)$$

By construction, β_{DA,s} is negative for procyclical sectors. Our calibration applies a sign inversion before mapping to CAPM-compatible scale.

D. Beaver-Kettler-Scholes Accounting Variables Beta

The BKS beta is the fitted value of a cross-sectional regression of Hill-Stone-estimated beta on observable accounting variables (sector median debt-to-equity, log of sector total assets, and intra-sector standard deviation of ROA):

$$\beta_{BKS,s} = \gamma_0 + \gamma_1 \cdot DE_s + \gamma_2 \cdot log(A_s) + \gamma_3 \cdot \sigma_{ROA,s} \quad (5)$$

E. Damodaran as External Benchmark

We source the most recent emerging-market unlevered beta estimates from Damodaran (2026) and map them to CIIU 2-digit sectors. The Damodaran beta is constant per sector across our sample period and serves as both an external comparability benchmark and an input (with 20% weight) to the Tier 4 fallback rule specification. We emphasize that Damodaran serves a comparability function for which it is well-suited; the CAB-Ke framework does not seek to displace this function but to complement it with locally informed sectoral discrimination.

V. VALIDATION, INFERENCE, AND EVIDENCE STANDARDS

A. Validation Arsenal

We apply 31 documented statistical tests organized in ten categories: out-of-sample point precision, forecast unbiasedness, efficiency and encompassing, directional precision, paired forecast comparison, information criteria, structural stability, cross-sectional macroeconomic validity, residual diagnostics, and Bayesian model comparison. Within the 31-test arsenal, five are non-discriminating in our sample and eight are mechanically redundant; the resulting discriminating subset comprises 18 tests, reported alongside the full 31-test score in Section VIII.

B. Multiple-Testing Correction

Per-sector inferential decisions involve 54 hypothesis tests (27 sectors × 2 component specifications). We apply the Benjamini-Hochberg (1995) false discovery rate correction to per-sector p-values used for Tier 1 classification. Raw uncorrected p-values are retained for Tier 2 and Tier 3 classifications, reflecting the operational reality that strict multiple-testing correction across 27 sectors in an emerging market would eliminate operationally usable evidence.

C. Academic versus Professional Evidence in Data-Constrained Markets

In a developed-market setting with extensive time-series data and traded market benchmarks, the conventional academic evidence standard requires p-values below 0.05 with multiple-testing correction across test families and 95% confidence intervals for inferential statements. This standard is appropriate where it can be met and is the

standard for academic claims in the finance literature. In an emerging-market setting with annual accounting frequency, short panels of typically T = 19 observations, and absent traded market betas for most firms, the academic standard cannot be met for most sectors regardless of methodological care. The framework must nonetheless produce defensible cost-of-equity estimates for professional valuation work.

CAB-Ke distinguishes three evidence standards, each with a specific professional use case:

(1) Strict Academic Evidence. Benjamini-Hochberg-corrected p-values below 0.05 on both component betas, with 95% confidence intervals and signs consistent with the procyclical convention. This standard supports academic claims of statistical validation. In our sample, only one sector qualifies: CIIU 70 Administrative Services. We report this transparently as the strict academic reference set.

(2) Strong Professional Evidence. Asymmetric criterion: BH-corrected p-value below 0.05 on the Hill-Stone factor AND below 0.10 on the distress-augmented factor, with correct sign and open-interval beta within economically reasonable bounds. The 90% confidence interval is the professional disclosure standard. This evidence level supports use as principal valuation input with mandatory tier-specific disclosure. In our sample, three sectors qualify: CIIU 6 Petroleum, CIIU 70 Administrative Services, CIIU 77 Rental and Leasing.

(3) Emerging-Market Indicative Evidence. Raw p-values below 0.20 on at least one component, with sign consistent with economic intuition, plausibility under sectoral structural characteristics, and consistency with the BKS specification, the sectoral median, or the Damodaran benchmark. This level supports indicative estimates with explicit disclosure that local evidence is weak. It is not academic validation and must not be presented as such.

Table 3

Evidence Standards: Academic vs Professional vs Indicative

Three distinct evidence standards underlie CAB-Ke's tier classification. Each standard is appropriate for a specific use case and must not be conflated with the others.

Standard	Statistical criterion	CI level	Professional use
A. Strict Academic	BH-FDR p<0.05 on BOTH HS AND DA components, positive signs	**95%**	*Academic claims of statistical validation. Our sample: 1 secto 70).*
B. Strong Professional	ASYMMETRIC: BH-FDR p_HS<0.05 AND p_DA<0.10, positive sign, β in range	**90%**	*Principal valuation input under Colombian constraints with m disclosure. Maps to Tier 1.*
C. EM Indicative	Raw p < 0.15 or < 0.20 on at least one component, positive sign, plausibility	**80-85%**	*Indicative use with explicit disclosure. Cannot be presented as to Tier 3.*

These three standards are kept conceptually distinct throughout the paper. We do not present professional evidence as academic validation, nor indicative evidence as professional standard. The three labels are mutually exclusive; a sector is classified at the highest standard for which its evidence qualifies. Tier 1 of the reliability classification (Section VI) corresponds to Strong Professional Evidence and explicitly is NOT equivalent to strict symmetric dual-factor academic validation; only one sector meets the strict symmetric academic criterion.

VI. THE FIVE-TIER RELIABILITY CLASSIFICATION

The central organizing principle of the framework is the five-tier reliability classification. Each of the 27 non-financial sectors plus the 2 excluded financial sectors is assigned to one of five tiers based on the strength of local empirical evidence and the applicability of framework assumptions. The five tiers are mutually exclusive; a sector is assigned to the highest tier for which its evidence qualifies.

(1) Tier 1 — Strong Professional Evidence. BH-corrected Hill-Stone p-value below 0.05 AND BH distress-augmented p below 0.10, signs consistent with the procyclical convention, and open-interval beta within [0.30, 2.50]. β_U from open-interval Vasicek combination. Professional use: principal beta for valuation with mandatory disclosure of confidence interval, Damodaran benchmark, and sensitivity. Three sectors qualify: CIIU 6 Petroleum, CIIU 70 Administrative Services, CIIU 77 Rental and Leasing. The criterion is asymmetric and NOT equivalent to strict symmetric dual-factor academic validation, which only one sector (CIIU 70) attains.

(2) Tier 2 — Professional Use. At least one component achieves raw p-value below 0.10 with correct sign and beta within reasonable bounds. β_U from open-interval Vasicek combination. Professional use: principal beta for professional valuation with mandatory sensitivity disclosure. Six sectors qualify: CIIU 9, 22, 25, 46, 47, 55. This evidence level does NOT constitute strict academic validation and must not be presented as such.

(3) Tier 3 — Indicative. Raw p-value below 0.20 on at least one component with correct sign, but neither achieves the Tier 2 operational threshold. β_U from additional Bayesian shrinkage toward CAPM-neutral prior with tighter σ. Professional use: indicative beta with shrinkage; explicit comparison against Damodaran, BKS, and sectoral median is required. Two sectors qualify: CIIU 17 Paper, CIIU 82 Administrative Support Services.

(4) Tier 4 — Fallback. Local time-series evidence is insufficient. β_U from the blend 0.50·β_BKS + 0.30·β_median + 0.20·β_Damodaran (baseline weights). Professional use: fallback beta only with analyst override and explicit disclosure that local evidence is insufficient; the three component betas are reported separately. Sixteen sectors qualify. Each Tier 4 sector includes an explicit Fallback Reason documenting why local evidence is insufficient (typically sign inconsistency, implausibly low open-interval beta, or insufficient raw p-values).

(5) Tier 5 — Special Treatment. Framework applicability assumptions are not met. SIREM's D/E does not capture customer-deposit leverage for financial intermediaries, making Hamada inapplicable. The framework does not produce a Ke estimate; a separate framework based on traded bank comparables is recommended. Two sectors qualify: CIIU 64 Financial Services, CIIU 66 Auxiliary Financial Services.

Table 4
Five-Tier Reliability Classification Summary

The five tiers are mutually exclusive. A sector is assigned to the highest tier for which its evidence qualifies. Classification governs which estimation rule applies (open-interval Vasicek, additional shrinkage, or fallback blend) and which disclosure requirements apply.

Tier	N	Criterion	Final β rule	Sectors

1. Strong Professional Evidence	**3**	BH p_HS<0.05 AND BH p_DA<0.10; positive sign; β in [0.30, 2.50]	β_open from open-interval Vasicek with CAPM-neutral prior	*CIIU 6 Petroleum; CIIU 70 Rental*
2. Professional Use	**6**	Raw p < 0.10 on at least one component; positive sign; β in range	β_open from open-interval Vasicek	*CIIU 9, 22, 25, 46, 47, 55*
3. Indicative	**2**	Raw p < 0.20 on at least one component; positive sign; β in range	Additional Bayesian shrinkage; tighter prior (σ=0.30)	*CIIU 17 Paper; CIIU 82 Ad*
4. Fallback	**16**	Insufficient evidence: sign issue, implausible β, or raw p > 0.20	0.50·BKS + 0.30·median + 0.20·Damodaran	*CIIU 1, 10, 13, 14, 20, 21, 2 45, 52, 68, 71, 74*
5. Special Treatment	**2**	Framework assumptions not met (D/E excludes deposit leverage)	Not applicable; use separate framework	*CIIU 64, CIIU 66 (financial*

VII. THE CAB-KE ESTIMATION PROCEDURE

The procedure is organized into five layers. Layers 1 and 2 produce raw component betas and their combination. Layer 3 applies open-interval Vasicek shrinkage with a CAPM-neutral prior, which is the framework's primary calibration. Layer 4 applies tier-specific estimation rules. Layer 5 produces cost of equity in U.S. dollars and Colombian pesos.

A. Layer 1: Component Beta Estimation

Hill-Stone and distress-augmented betas estimated as in Section IV, equations (3) and (4). Inference by stationary bootstrap (L = 4, 1,000 replicates). Raw p-values are recorded; Benjamini-Hochberg-corrected p-values are computed for Tier 1 classification.

B. Layer 2: Combination

Inverse-variance and GLS combinations:

$$\beta_{IV,s} = w_{HS,s} \cdot \beta_{HS,s} + (1 - w_{HS,s}) \cdot \beta_{DA,s} \quad (6)$$

$$w_{HS,s} = SE_{HS,s}^{-2} / (SE_{HS,s}^{-2} + SE_{DA,s}^{-2}) \quad (7)$$

$$w_{GLS} \propto \Sigma^{-1} \cdot 1, \text{ where } \Sigma \text{ models residual correlation } \rho \approx 0.45 \quad (8)$$

Inverse-variance is the minimum-variance combination under residual independence. As both component betas are estimated on the same sector-year panel, residual independence is unlikely; GLS combination is preferable where IV and GLS combinations diverge by more than 0.10. In our sample, the median absolute IV-GLS divergence is 0.18, with the maximum at 1.39 in CIIU 9.

C. Layer 3: Open-Interval Vasicek with CAPM-Neutral Prior

The combined raw beta is shrunk toward the CAPM-neutral prior β_0 = 1.0 with prior standard deviation σ_0 = 0.40:

$$\beta_{open,s} = (\beta_{IV,s}/SE_{IV,s}^2 + \beta_0/\sigma_0^2) / (1/SE_{IV,s}^2 + 1/\sigma_0^2) \quad (9)$$

The CAPM-neutral prior reflects the principle that, absent strong evidence, the prior expectation for any sector beta is 1.0. We report the framework under this open-interval Vasicek specification as the headline. The capped specification within the ranking-anchored interval [0.40, 1.60] is reported as a robustness alternative. The Spearman

rank correlation between open-interval and capped specifications is 0.955; however, the capped specification flags 15 of 27 sectors at the boundary, 13 of 27 with effective change, and 8 of 27 with material change above 0.10. The open-interval specification is preferred as headline because it avoids artificial compression for sectors with strong evidence at either extreme.

D. Layer 4: Tier-Specific Final Beta

The final unlevered beta is determined by tier:

(1) Tiers 1 and 2: $\beta_{U,s}^{CAB} = \beta_{open,s}$ (open-interval Vasicek posterior).

(2) Tier 3: Additional Bayesian shrinkage toward $\beta_0 = 1.0$ with tighter $\sigma_0 = 0.30$.

(3) Tier 4: Fallback blend $\beta_{U,s}^{CAB} = 0.50 \cdot \beta_{BKS,s} + 0.30 \cdot \bar{\beta} + 0.20 \cdot \beta_{Damo,s}$ (baseline weights). The framework also documents a quality-score weighted alternative described in equation (10): $w_j = Q_j / \Sigma Q_j$ where Q_j depends on data coverage, standard error, distance to calibration bounds, stability under sensitivity, predictive score, economic consistency, and benchmark availability. The quality-score implementation requires additional analyst inputs and is reserved for future revision; the baseline fixed weights are the operational default.

$$\beta_{fb,s}^{Q} = \Sigma_j \, w_{j,s} \cdot \beta_{j,s} \text{ where } w_{j,s} = Q_{j,s} \,/\, \Sigma_k \, Q_{k,s} \quad (10)$$

E. Layer 5: Cost of Equity Conversion

Hamada relevering (1972) with Colombian standard corporate tax rate $\tau = 35\%$ (Ley 2277/2022) as headline:

$$\beta_{L,s} = \beta_{U,s}^{CAB} \cdot [1 + (1 - \tau) \cdot D_s/E_s] \quad (12)$$

Pereiro Adjusted Hybrid CAPM in USD:

$$Ke_{USD,s} = R_f^{USD} + CRP_{COL} + \beta_{L,s} \cdot ERP_{US} \quad (13)$$

Fisher conversion to Colombian pesos with long-run inflation expectations as headline:

$$Ke_{COP,s} = (1 + Ke_{USD,s}) \cdot (1 + \pi_{COL}^{LR}) \,/\, (1 + \pi_{US}^{LR}) - 1 \quad (14)$$

Macroeconomic parameters (May 2026 snapshot — update at valuation date): $R_f^{USD} = 4.20\%$ (Fed H.15 10Y Treasury); $CRP_{COL} = 2.85\%$ headline (Damodaran), sensitivity range to 3.05% (CDS-implied) and 3.30% (EMBI+ adjusted); $ERP_{US} = 4.70\%$ (Damodaran); $\pi_{COL}^{LR} = 3.0\%$ (BanRep target); $\pi_{US}^{LR} = 2.3\%$ (Fed long-run). Sector-specific effective tax rates are reported in sensitivity in Section X.

VIII. EMPIRICAL RESULTS

A. Tier Distribution

The five-tier classification produces: three sectors in Tier 1 Strong Professional Evidence (CIIU 6, 70, 77); six sectors in Tier 2 Professional Use (CIIU 9, 22, 25, 46, 47, 55); two sectors in Tier 3 Indicative (CIIU 17, 82); sixteen sectors in Tier 4 Fallback; and two financial sectors in Tier 5 Special Treatment (CIIU 64, 66). The range of Ke COP under the long-run inflation specification runs from 10.90% (CIIU 82, Tier 3) to 20.17% (CIIU 6, Tier 1), with a median of 13.83% across the 27 sectors.

Table 5

Tier Distribution Across Colombian Sectors

Distribution of 29 sectors (27 non-financial + 2 financial) across the five reliability tiers. The framework analyzes only the 27 non-financial sectors; the 2 financial sectors are classified as Tier 5 Special Treatment and excluded from the matrix because SIREM's accounting D/E does not capture deposit leverage.

Tier	N	Notes
Tier 1 — Strong Professional Evidence	3	Asymmetric BH p<0.05 (HS) AND p<0.10 (DA). Of these, only CIIU 70 meets the strict symmetric dual p<0.05 criterion.
Tier 2 — Professional Use	6	Raw p < 0.10 on at least one component with positive sign and β in range.
Tier 3 — Indicative	2	Raw p < 0.20 on at least one component; additional Bayesian shrinkage applied.
Tier 4 — Fallback	16	Insufficient local evidence; β from baseline blend 0.50·BKS + 0.30·median + 0.20·Damodaran.
Tier 5 — Special Treatment	2	CIIU 64 and 66 (financial intermediation). Excluded from matrix; separate methodology required.

B. Strict Academic Reference Set

Under the strict symmetric academic criterion of BH-corrected p-value below 0.05 on BOTH Hill-Stone and distress-augmented components with sign consistency, only one sector qualifies: CIIU 70 Administrative Services. This strict academic reference set is reported here as a transparency disclosure to support academic claims of statistical validation; it is not used for tier classification. The framework's Tier 1 classification uses an asymmetric criterion (BH p<0.05 on Hill-Stone, BH p<0.10 on distress-augmented) that captures the three sectors with the strongest local evidence within Colombian data constraints, balancing inferential rigor against operational usefulness for Colombian valuation practice.

C. Methodological Orthogonality with Damodaran

The Spearman rank correlation between CAB-Ke's open-interval beta and the Damodaran benchmark across the 27 non-financial sectors is approximately −0.20. Orthogonality is the paper's central substantive finding. The two methodologies serve complementary functions: Damodaran provides international comparability across transactions calibrated on average emerging-market sectoral composition; CAB-Ke provides sector discrimination within Colombia calibrated on local accounting evidence. The orthogonality is robust across all robustness specifications reported in Section X.

D. Fallback Reasons

Each Tier 4 sector includes an explicit Fallback Reason in the matrix. The sixteen Tier 4 sectors decompose into three not-mutually-exclusive reason categories.

(1) Insufficient raw p-values (seven sectors). CIIU 10, 13, 14, 20, 21, 23, 71 fail the operational threshold on both components with raw p > 0.20. These sectors are Fallback because time-series evidence does not support a sector-specific estimate.

(2) Wrong sign on Hill-Stone (six sectors). CIIU 27, 41, 43, 45, 52, 74 exhibit negative β_HS_raw, failing the procyclical convention. These sectors typically also have implausibly low β_open (below 0.30) compounding

the issue. The negative slope is economically inconsistent with the expectation that sectoral profitability rises with aggregate market profitability.

(3) Borderline cases with implausibly low β_open (three sectors). CIIU 1 Agriculture (β_open = 0.151), CIIU 42 Civil Engineering (β_open = 0.156), and CIIU 68 Real Estate (β_open = 0.266). CIIU 68 is the most notable case: both raw p-values are below 0.10 with correct sign, but β_open is marginally below the 0.30 lower bound. We classify all three as Tier 4 with explicit BORDERLINE disclosure in the fallback reason. Professionals may, with analyst override and additional disclosure, treat CIIU 68 as Tier 2 under a relaxed lower bound.

Total Tier 4 count is 16. Some sectors satisfy more than one fallback reason (typically sign issue combined with implausibly low β); the framework records all applicable reasons in the matrix.

IX. SECTOR-LEVEL COST-OF-EQUITY IMPLEMENTATION

This section presents the complete cost-of-equity matrix for the 27 non-financial Colombian sectors. Table 6 provides the matrix with all sector parameters. Columns include: CIIU code, sector name, tier, tier rationale, component betas (β_HS, β_DA, β_BKS), separate calibration outputs (β_open, β_capped, β_shrunk, β_fallback), final CAB-Ke beta (β_CAB_final), 90% and 95% confidence intervals, Damodaran benchmark, gap percentage, sector median D/E, sector-specific tax, Hamada-levered beta, USD cost of equity, COP cost of equity under long-run and spot inflation, recommended professional use, required disclosure, and—for Tier 4 sectors—the Fallback Reason.

Table 6
CAB-Ke Sectoral Cost-of-Equity Matrix — Final Static Results

The 27 non-financial sectors of the CAB-Ke Final Static Results Matrix with tier classification, final unlevered beta from open-interval Vasicek specification, Hamada-levered beta at τ = 35% standard, Ke COP under long-run Fisher conversion, Damodaran benchmark, and gap percentage. The full 35-column matrix including β_open, β_capped, β_shrunk, β_fallback, CIs, sector-specific tax rates, spot Ke COP, tier rationale, recommended use, required disclosure, and Fallback Reason is in the supplementary Excel file 'CAB_Ke_Matrix.xlsx (Final Static Results Matrix)'. CIIU 64 and 66 (Tier 5) are excluded.

CIIU	Sector	Tier	β_U CAB	β_L τ=35%	Ke COP LR	β Damo	Gap %	Rationale
70	Admin support services	T1	**0.61**	0.74	**11.28%**	*1.05*	**-42.2%**	*BH p_HS<0.05 + BH p_DA<0.10; asymmetric criterion*
77	Rental/leasing	T1	**0.55**	0.98	**12.43%**	*0.85*	**-35.7%**	*BH p_HS<0.05 + BH p_DA<0.10; asymmetric criterion*
6	Petroleum & gas	T1	**1.79**	2.62	**20.17%**	*0.92*	**+94.8%**	*BH p_HS<0.05 + BH p_DA<0.10; asymmetric criterion*
9	Mining support	T2	**1.01**	1.50	**14.87%**	*1.15*	-11.8%	*Raw p < 0.10 on one component + sign + range*
22	Rubber/plastic	T2	**0.40**	0.67	**10.96%**	*0.92*	**-56.0%**	*Raw p < 0.10 on one component + sign + range*
25	Metal products	T2	**0.63**	1.10	**13.01%**	*1.10*	**-43.1%**	*Raw p < 0.10 on one component + sign + range*
46	Wholesale trade	T2	**0.38**	0.67	**10.97%**	*0.95*	**-60.3%**	*Raw p < 0.10 on one component + sign + range*
47	Retail trade	T2	**0.47**	0.69	**11.05%**	*0.90*	**-47.4%**	*Raw p < 0.10 on one component + sign + range*
55	Accommodation	T2	**0.75**	1.10	**12.99%**	*1.15*	**-35.1%**	*Raw p < 0.10 on one component + sign + range*
17	Paper	T3	**0.46**	0.66	**10.92%**	*0.88*	**-47.9%**	*Raw p < 0.20; indicative only*
82	Admin services	T3	**0.49**	0.66	**10.90%**	*0.95*	**-48.8%**	*Raw p < 0.20; indicative only*
1	Agriculture	T4	**0.77**	1.01	**12.59%**	*0.78*	-0.8%	*Insufficient signal; fallback blend (see Excel)*

10	Food	T4	**0.75**	1.24	**13.66%**	*0.71*	+6.1%	*Insufficient signal; fallback blend (see Excel)*
13	Textiles	T4	**0.79**	1.28	**13.83%**	*0.95*	-16.8%	*Insufficient signal; fallback blend (see Excel)*
14	Apparel	T4	**0.76**	1.50	**14.89%**	*0.95*	-20.3%	*Insufficient signal; fallback blend (see Excel)*
20	Chemicals	T4	**0.85**	1.36	**14.24%**	*0.95*	-10.1%	*Insufficient signal; fallback blend (see Excel)*
21	Pharmaceuticals	T4	**0.87**	1.33	**14.05%**	*1.05*	-17.2%	*Insufficient signal; fallback blend (see Excel)*
23	Non-metallic minerals	T4	**0.81**	1.28	**13.86%**	*0.98*	-17.6%	*Insufficient signal; fallback blend (see Excel)*
27	Electrical equipment	T4	**0.81**	1.46	**14.70%**	*1.05*	-22.6%	*Insufficient signal; fallback blend (see Excel)*
41	Building construction	T4	**0.76**	1.46	**14.70%**	*1.05*	-27.2%	*Insufficient signal; fallback blend (see Excel)*
42	Civil engineering	T4	**0.82**	1.29	**13.90%**	*1.10*	-25.4%	*Insufficient signal; fallback blend (see Excel)*
43	Specialized construction	T4	**0.80**	1.32	**14.05%**	*1.10*	-27.7%	*Insufficient signal; fallback blend (see Excel)*
45	Vehicle trade	T4	**0.79**	1.42	**14.50%**	*1.05*	-24.3%	*Insufficient signal; fallback blend (see Excel)*
52	Storage	T4	**0.85**	1.40	**14.42%**	*1.05*	-18.6%	*Insufficient signal; fallback blend (see Excel)*
68	Real estate	T4	**0.73**	0.96	**12.31%**	*0.65*	+12.6%	*Insufficient signal; fallback blend (see Excel)*
71	Architecture/engineering	T4	**0.84**	1.53	**15.04%**	*1.05*	-19.9%	*Insufficient signal; fallback blend (see Excel)*
74	Professional services	T4	**0.87**	1.04	**12.69%**	*1.00*	-13.0%	*Insufficient signal; fallback blend (see Excel)*

Notes: Sectors ordered by tier (1-4) then by CIIU within tier. β_U CAB is the final CAB-Ke unlevered beta computed by tier-specific rule (open-interval Vasicek for Tiers 1-2; additional shrinkage for Tier 3; fallback blend 0.50·BKS + 0.30·median + 0.20·Damodaran for Tier 4). β_L applies Hamada with τ = 35% standard. Ke COP LR uses Fisher conversion with long-run inflation expectations (π_COL = 3.0%, π_US = 2.3%). Gaps > 30% (absolute) highlighted in red. Macro parameters (May 2026): Rf = 4.20%, CRP = 2.85%, ERP = 4.70%; update at valuation date.

Table 7

Sectors with Large Absolute Gaps versus Damodaran (>30%)

Sectors where the absolute gap between final CAB-Ke beta and the Damodaran emerging-market benchmark exceeds 30%. Large gaps do not imply that CAB-Ke is methodologically superior; they signal that CAB-Ke captures a different sensitivity from the international benchmark. The two methodologies serve complementary functions: Damodaran provides external comparability; CAB-Ke provides locally informed sectoral discrimination. The PROFESSIONAL RULE on large gaps appears in Sections IX and XI: when |Gap| > 30%, the valuation report must include mandatory sensitivity with Damodaran benchmark and, where appropriate, a midpoint scenario.

CIIU	Sector	Tier	β CAB	β Damo	Gap	Interpretation
6	Petroleum & gas	**Tier 1**	**1.79**	*0.92*	**+94.8%**	*Greater local cyclical sensitivity than international peers; r petroleum concentration in mature fields.*
70	Admin support services	**Tier 1**	**0.61**	*1.05*	**-42.2%**	*Lower local cyclical sensitivity than international peers; Co outsourcing dominated by long-term BPO contracts.*
77	Rental & leasing	**Tier 1**	**0.55**	*0.85*	**-35.7%**	*Lower local cyclical sensitivity; equipment rental linked to cycle but with longer-duration contracts.*
22	Rubber & plastic	**Tier 2**	**0.40**	*0.92*	**-56.0%**	*Local evidence shows lower cyclicality; benchmark domina international peers linked to petrochemicals.*
25	Metal products	**Tier 2**	**0.63**	*1.10*	**-43.1%**	*Local metal-mechanic sector less cyclical than internationa domestic-oriented production.*
46	Wholesale trade	**Tier 2**	**0.38**	*0.95*	**-60.3%**	*Markedly lower local cyclicality; Colombian wholesale less cyclical durables than international comparable.*
47	Retail trade	**Tier 2**	**0.47**	*0.90*	**-47.4%**	*Lower local cyclicality; Colombian modern retail expansio reflected stable demand patterns.*
55	Accommodation	**Tier 2**	**0.75**	*1.15*	**-35.1%**	*Lower local cyclicality; Colombian hotel sector dominated rather than discretionary tourism.*
17	Paper	**Tier 3**	**0.46**	*0.88*	**-47.9%**	*Indicative only; Colombian paper sector small and locally*

82	Admin support services	**Tier 3**	**0.49**	*0.95*	**-48.8%**	*Indicative only; local services less cyclical than internation*

Professional Rule for Large Gaps.

When the absolute gap between CAB-Ke and Damodaran exceeds 30%, the valuation report must include mandatory sensitivity with Damodaran benchmark and, where appropriate, a midpoint scenario. This rule applies regardless of the sector's tier classification: even Tier 1 sectors with large gaps must be benchmarked against the international reference for analyst review. The rule reflects the principle that CAB-Ke and Damodaran capture complementary functions—local sensitivity and international comparability—and that large divergences merit transparent disclosure rather than methodological dismissal.

The full matrix is provided in the supplementary Excel file alongside additional sheets: Open-vs-Cap sensitivity, CRP sensitivity across three sources, and Tier 4 Fallback detail with fallback reasons. The supplementary Excel file reports the final static results matrix. A dynamic professional calculator is a recommended implementation extension and is not included in the current supplementary file.

X. ROBUSTNESS AND SENSITIVITY ANALYSIS

A. Open-Interval vs Capped Vasicek

The capped specification flags 15 of 27 sectors at the [0.40, 1.60] boundary, with 13 of 27 sectors exhibiting effective change between open and capped, and 8 of 27 exhibiting material change above 0.10. The Spearman rank correlation between specifications is 0.955, indicating that overall ranking is largely preserved despite material changes in specific sector point estimates. The open-interval specification is the headline. Professionals requiring CAPM-compatible scale range (e.g., regulatory presentations) may report the capped estimate as alternative.

B. Multiple p-Value Thresholds

Under $p < 0.05$ (BH-corrected): only one sector qualifies as Tier 1 under strict symmetric criterion (CIIU 70). Under $p < 0.10$ (asymmetric BH): three sectors qualify as Tier 1. Under raw $p < 0.10$: six additional sectors qualify as Tier 2. Under raw $p < 0.15$: ten sectors qualify cumulatively. Under raw $p < 0.20$: eleven sectors qualify cumulatively. The framework acknowledges that $p < 0.05$ with BH correction is strict academic; raw $p < 0.10$ is operational professional; raw $p < 0.20$ is emerging-market indicative evidence.

C. Confidence Interval Comparison

All point estimates are accompanied by both 90% and 95% confidence intervals in the matrix. The 90% CI is the professional disclosure standard under Tier 2 classification; the 95% CI is the academic reference. For Tier 1 estimates both are reported. The CI width under posterior Bayesian SE varies from approximately 0.18 (CIIU 70 with strongest evidence) to 0.78 (Tier 3 sectors with weaker evidence). Tier 4 Fallback estimates use a conservative SE of 0.20.

D. Sensitivity to Country Risk Premium

Under three documented CRP sources—Damodaran (2.85%, headline), CDS-implied (3.05%), and EMBI+ adjusted (3.30%)—the Ke COP estimates shift approximately 45 basis points per sector across the range. The supplementary Excel includes a dedicated CRP sensitivity sheet with the full 27×3 matrix of Ke COP estimates by source.

E. Spot versus Long-Run Inflation

Under spot inflation (π_{COL} = 5.4%, π_{US} = 2.5%, April 2026 snapshot), Fisher conversion produces Ke COP estimates approximately 280 basis points above those under long-run inflation expectations (π_{COL} = 3.0%, π_{US} = 2.3%). Short-horizon valuations may prefer the spot specification; multi-year DCF valuations should use the long-run specification consistent with the going-concern assumption.

F. Tax Rate Sensitivity

The headline matrix uses the Colombian standard corporate tax rate τ = 35% (Ley 2277/2022). Sensitivity is reported under sector-specific effective rates: 15% for agriculture (Ley 1429 primary-production exemptions), 20% for pharmaceuticals in free trade zones, and 40% for petroleum (with surcharge). Sector-specific rates shift β_L approximately ±10% relative to the standard rate; firm-specific effective rates may apply where the valuation mandate permits firm-level tax analysis.

G. Sensitivity to D/E

Hamada relevering uses the sector median D/E ratio. For valuation of a specific firm, the professional should use the firm's target D/E rather than the sector median. The Excel matrix supports this sensitivity by allowing direct D/E override in the Hamada formula; β_L and Ke recalculate automatically. For firms with materially different D/E from the sector median, this sensitivity may shift Ke by 200-400 basis points.

H. Fallback Rule Weight Sensitivity

The Tier 4 fallback blend uses fixed weights 0.50 BKS + 0.30 median + 0.20 Damodaran as baseline. See Section X.M below for a comprehensive sensitivity analysis under five alternative weight specifications (Equal weights, BKS-dominant, Median-dominant, Damodaran-emphasized, and BKS-only).

I. Inverse Variance versus GLS Combination

The IV-GLS divergence has median 0.18 and maximum 1.39 (CIIU 9). For Tier 1 and Tier 2 sectors, the divergence is below 0.10 in most cases; any combination is defensible. For Tier 3 and Tier 4 sectors, the divergence is irrelevant because the final beta is computed via shrinkage or fallback blend rather than direct combination.

J. EMBI-Based Distress Factor

As an alternative to the Z-score-based distress factor (subject to look-ahead bias because Z-score parameters were calibrated on data 2018-2024), we recompute the distress-augmented beta using year-over-year changes in the EMBI+ Colombia spread. The EMBI-based factor is completely out-of-sample relative to the Colombian accounting panel. Under this alternative, the distress-augmented beta exhibits rank correlation of 0.78 with the Z-score-based specification. The Tier 1 classification (CIIU 6, 70, 77) is preserved under both specifications.

K. Naïve Benchmarks

The framework's predictive performance is evaluated against three naïve benchmarks: cross-sectional median, AR(1) on sector ROA changes, and aggregate ΔROA_M. The inverse-variance combination's out-of-sample R-squared is +0.30 against median, +0.18 against AR(1), and +0.12 against ΔROA_M. The framework's dominance is preserved across all naïve benchmarks.

L. Prior Sensitivity Analysis

The open-interval Vasicek specification with $\beta_0 = 1.0$ and $\sigma_0 = 0.40$ enters the framework at Layer 3 (Section VII.C), where it shrinks the inverse-variance combined beta toward the prior mean with strength governed by the prior precision. This shrinkage affects the final unlevered beta estimate $\beta_{U,s}^{CAB}$ for Tier 1-2 sectors—where the open-interval Vasicek is the active estimation rule—but does not affect the tier classification itself, which depends on the raw bootstrap p-values from Layer 1. We therefore test the sensitivity of the final β_U^{CAB} and the resulting Ke COP, across the nine Tier 1-2 sectors, to six alternative prior specifications.

The alternative priors are: (i) $\beta_0 = 0.8$, $\sigma_0 = 0.40$ (pessimistic mean prior); (ii) $\beta_0 = 1.2$, $\sigma_0 = 0.40$ (optimistic mean prior); (iii) $\beta_0 = 1.0$, $\sigma_0 = 0.30$ (tight precision prior); (iv) $\beta_0 = 1.0$, $\sigma_0 = 0.60$ (loose precision prior); (v) Empirical Bayes specification, $\beta_0^{EB} = \text{median}(\beta_{HS,s}) \approx 0.75$ and $\sigma_0^{EB} = 1.4826 \times \text{MAD}(\beta_{HS,s}) \approx 0.32$, with MAD denoting the median absolute deviation, both estimated from the cross-section of our own Hill-Stone beta sample; and (vi) Jeffreys reference prior with $\sigma_0 \to \infty$ (improper uninformative prior), under which the posterior collapses to the inverse-variance estimator with no shrinkage. Table 8 reports the median and maximum absolute change in β_U^{CAB} and Ke COP across the nine Tier 1-2 sectors under each alternative prior, relative to the baseline.

Table 8

Prior Sensitivity: Impact of Alternative Vasicek Priors on β_U^{CAB} and Ke COP

Sensitivity of the final unlevered beta and Ke COP under six alternative Vasicek prior specifications, computed for the nine Tier 1-2 sectors where the open-interval Vasicek is the active estimation rule. Changes are reported as median and maximum absolute differences relative to the baseline ($\beta_0 = 1.0$, $\sigma_0 = 0.40$). The Empirical Bayes prior is estimated from the cross-section of Hill-Stone betas in the same sample; the Jeffreys reference is an improper uninformative prior corresponding to no shrinkage.

Prior specification	β_0	σ_0	Median $\lvert\Delta\beta_U\rvert$	Max $\lvert\Delta\beta_U\rvert$	Median $\lvert\Delta Ke\rvert$ (pp)	Max
Baseline (open-interval Vasicek)	1.00	0.40	—	—	—	—
(i) Pessimistic mean	0.80	0.40	0.06	0.14	0.36	0.96
(ii) Optimistic mean	1.20	0.40	0.06	0.13	0.36	0.89
(iii) Tight precision	1.00	0.30	0.09	0.21	0.53	1.44
(iv) Loose precision	1.00	0.60	0.02	0.05	0.12	0.34
(v) Empirical Bayes	0.75	0.32	0.07	0.18	0.41	1.24
(vi) Jeffreys (no shrinkage)	—	∞	0.15	0.43	0.89	2.95

Three findings emerge. First, under the four parametric alternatives (i)-(iv), the median absolute change in β_U^{CAB} is at most 0.09 (under the tight precision prior), corresponding to a median absolute change in Ke COP of 0.53 percentage points. Maximum changes are concentrated in CIIU 6 Petroleum, the sector with the largest raw

beta (1.79) and thus most affected by shrinkage adjustments. The directional pattern is consistent with intuition: under pessimistic mean priors (β_0 = 0.8) high-beta sectors shift downward; under tight precision priors (σ_0 = 0.30) all sectors shift toward β_0 = 1.0.

Second, the Empirical Bayes specification (v) yields β_0^{EB} ≈ 0.75 and σ_0^{EB} ≈ 0.32, both consistent with the empirical concentration of Colombian sector betas below the CAPM-neutral 1.0. Under this prior, the median absolute change in β_U^{CAB} is 0.07 (max 0.18, CIIU 6), with a median absolute change in Ke COP of 0.41 percentage points (max 1.24 pp). The Empirical Bayes specification is internally consistent (it uses the data to estimate the prior) but yields somewhat lower Ke COP estimates due to the lower β_0^{EB} ≈ 0.75. This represents an attractive complementary specification for analysts who prefer fully data-driven priors and is reported as a robustness alternative in the supplementary Excel file.

Third, the Jeffreys reference prior (vi) produces no shrinkage and recovers the raw inverse-variance combination. The median absolute change in β_U^{CAB} is 0.15 (max 0.43, CIIU 6), corresponding to a median absolute change in Ke COP of 0.89 pp (max 2.95 pp, CIIU 6). These are economically meaningful changes but do not alter qualitative rankings: the Spearman rank correlation between baseline and Jeffreys Ke COP across the nine Tier 1-2 sectors is 0.97, and the same three sectors (CIIU 6, 70, 77) remain the highest-evidence Tier 1 cases regardless of prior.

These results establish that the framework's quantitative estimates are robust under reasonable variations in the Vasicek prior specification. The baseline (β_0 = 1.0, σ_0 = 0.40) aligns with both theoretical motivation (CAPM-neutral default) and empirical evidence (cross-sectional dispersion of emerging-market betas in Damodaran 2026). Practitioners should report the baseline specification as primary; for Tier 1-2 valuations subject to elevated scrutiny, the Empirical Bayes alternative provides an internally consistent robustness disclosure.

M. Tier 4 Combination Weight Sensitivity (Comprehensive)

Section X.H introduced the Tier 4 fallback blend β_U^{Fallback} = 0.50·β_{BKS} + 0.30·β̄ + 0.20·β_{Damo} as the baseline weight specification. We now report a comprehensive sensitivity analysis under five alternative weight specifications, computed across the sixteen Tier 4 sectors. The headline weights 0.50/0.30/0.20 reflect the relative cross-sectional informativeness of each component, as discussed in Section IV.A.

The alternative weight schemes are: (i) Equal weights, 1/3 BKS, 1/3 median, 1/3 Damodaran; (ii) BKS-dominant, 0.70/0.20/0.10; (iii) Median-dominant, 0.30/0.50/0.20; (iv) Damodaran-emphasized, 0.40/0.20/0.40; (v) BKS-only, 1.00/0.00/0.00. For each scheme, we recompute β_U^{Fallback} for the sixteen Tier 4 sectors and convert to Ke COP via Hamada → Pereiro → Fisher. Table 9 reports the median and maximum absolute change in Ke COP relative to the baseline.

Table 9

Tier 4 Combination Weight Sensitivity: Impact on Ke COP

Sensitivity of Ke COP for the sixteen Tier 4 sectors under five alternative weight specifications for the fallback blend β_U^{Fallback} = w_{BKS}·β_{BKS} + w_{med}·β̄ + w_{Damo}·β_{Damo}. Changes are reported as median and maximum absolute differences in Ke COP (percentage points) relative to the baseline 0.50/0.30/0.20 weights. The maximum change column identifies the sector exhibiting the largest deviation under each scheme.

Scheme	w_{BKS}	w_{med}	w_{Damo}	Median |ΔKe| (pp)	Max |ΔKe| (pp)	Sector at max
Baseline (headline)	0.50	0.30	0.20	—	—	—
(i) Equal weights	1/3	1/3	1/3	0.42	0.98	*CIIU 68 Real estate*
(ii) BKS-dominant	0.70	0.20	0.10	0.65	1.42	*CIIU 41 Building co*
(iii) Median-dominant	0.30	0.50	0.20	0.38	0.87	*CIIU 1 Agriculture*
(iv) Damo-emphasized	0.40	0.20	0.40	0.91	1.78	*CIIU 21 Pharmaceu*
(v) BKS-only	1.00	0.00	0.00	1.18	2.31	*CIIU 68 Real estate*

Under alternative schemes, the median absolute change in Ke COP across the sixteen Tier 4 sectors ranges from 0.38 percentage points (Median-dominant) to 1.18 percentage points (BKS-only). The maximum sector-specific change is 2.31 pp (BKS-only, CIIU 68 Real estate, where the BKS prediction is materially lower than the Damodaran benchmark). All alternative weight schemes preserve the tier classification: no Tier 4 sector would re-classify to a different tier under any alternative, since Tier 4 status depends on raw p-values from Layer 1 rather than combination weights from Layer 4.

The Damodaran-emphasized specification (iv) produces the largest upward shift in Ke COP (median +0.91 pp, max +1.78 pp, CIIU 21 Pharmaceuticals), reflecting Damodaran's higher beta estimates for many Colombian sectors compared to the local BKS predictions. The BKS-only specification (v) produces the largest downward shifts in extreme cases (CIIU 68 Real estate), reflecting the BKS prediction that Colombian real estate has a substantially lower beta than the international comparable. These sensitivities are economically modest in median but substantial in tail cases.

We recommend that practitioners using Tier 4 estimates report Ke COP under the headline 0.50/0.30/0.20 weights as the primary estimate, with the Damodaran-emphasized alternative (iv) as a secondary disclosure for high-stakes applications involving substantial international comparability concerns, and the BKS-only alternative (v) as a stress test for sectors with very different local versus international characteristics (notably CIIU 68 Real estate and CIIU 1 Agriculture).

Table 10

Robustness and Sensitivity Summary

Summary of thirteen robustness and sensitivity tests reported in Section X. Each row identifies the test, the main empirical finding, and the implication for professional use of the CAB-Ke matrix.

Robustness test	Main finding	Implication for professional use
Open vs Capped Vasicek	ρ=0.955; 15/27 at cap; 13/27 effective change; 8/27 material change >0.10	*Open is headline; Capped reported as comparability sensiti*
p-value thresholds	Strict BH p<0.05 dual: 1 sector. Asymmetric BH: 3. Raw p<0.10: 9 cumulative. Raw p<0.20: 11.	*Strict supports academic claims; raw thresholds support op*
CI 90% vs 95%	CI 90% is professional disclosure standard; CI 95% is academic reference.	*Tier 1 reports both; Tiers 2-3 typically 90%.*
CRP sensitivity (3 sources)	Damodaran 2.85% to EMBI+ 3.30% shifts Ke COP by ~45bp per sector.	*Report Ke under at least two CRP sources in professional d*
Long-run vs spot inflation	Spot inflation raises Ke COP by ~280bp vs long-run.	*Long-run is headline for going-concern DCF; spot for shor*

Tax rate sensitivity	Standard 35% vs sector rates shift β_L by ±10%.	*Headline uses 35%; firm-specific tax when valuation mand*
Sector vs target D/E	Sector median vs firm target D/E can shift Ke by 200-400bp.	*Use firm target D/E when valuing a specific company.*
Fallback weight sensitivity	Rank correlation across alternative weights exceeds 0.94.	*Baseline 50/30/20 is robust; quality-score weights are futur*
IV vs GLS combination	Median |IV-GLS|=0.18; max=1.39 (CIIU 9).	*For Tier 1-2, any combination defensible; GLS preferred w*
EMBI distress factor	Rank correlation 0.78 with Z-score spec; Tier 1 preserved.	*Mitigates Z-score look-ahead bias; robustness check rather alternative.*
Naïve benchmarks	OOS R^2 +0.30 vs median, +0.18 vs AR(1), +0.12 vs ΔROA_M.	*Framework's dominance preserved across all naïve benchm*
Prior sensitivity (Vasicek)	Median |Δβ_U|=0.06-0.09 under parametric alternatives; 0.07 under Empirical Bayes; ρ_Spearman vs Jeffreys = 0.97.	*Baseline (β_0=1.0, σ_0=0.40) is robust; Empirical Bayes a disclosure.*
Tier 4 weights (comprehensive)	Median |ΔKe| 0.38-1.18 pp across 5 alt schemes; max 2.31 pp (BKS-only, CIIU 68).	*Headline 50/30/20 as primary; Damo-emphasized and BKS alternatives.*

XI. PROFESSIONAL USE OF CAB-KE IN COLOMBIAN VALUATION PRACTICE

This section documents the practical decision rule for Colombian valuation work. The framework is designed for use by investment bankers (M&A advisory, capital markets), valuation professionals (private firm valuation, fairness opinions, purchase price allocation), family offices, transfer pricing specialists, arbitration experts, legal disputes experts, and tax authorities. The decision rule is keyed to the tier classification in Section VI.

A. Tier-Specific Decision Rule

(1) Tier 1 Strong Professional Evidence (3 sectors). Use the CAB-Ke beta as principal estimate. Report Ke as point estimate with all macroeconomic parameters documented and updated to valuation date. The Damodaran benchmark is reported for international comparability. Disclosure: tier classification, BH-corrected p-values, 90% and 95% CIs, asymmetric criterion disclosure (NOT equivalent to strict academic validation).

(2) Tier 2 Professional Use (6 sectors). Use the CAB-Ke beta as principal estimate with mandatory disclosure: 90% confidence interval, Damodaran benchmark, gap percentage, CRP sensitivity (2.85% to 3.30%), and narrative justifying the sector choice. The valuation report must explicitly state that the estimate meets the operational professional standard but not strict academic validation.

(3) Tier 3 Indicative (2 sectors). Use the CAB-Ke beta with shrinkage toward CAPM-neutral prior and disclose that evidence is indicative. Report the Damodaran benchmark and BKS specification as comparable alternatives. Provide analyst narrative justifying indicative use.

(4) Tier 4 Fallback (16 sectors). Use the fallback blend only with analyst override and explicit disclosure that local time-series evidence is insufficient. Report the three component betas (BKS, median, Damodaran) separately. Reference the Fallback Reason in the matrix to explain why local evidence is insufficient for this specific sector.

(5) Tier 5 Special Treatment (2 financial sectors). The framework does not apply. Use a separate methodology based on traded Colombian bank comparables or international financial-sector comparables with country-risk adjustment.

Table 11
Professional Decision Rule

Tier-specific decision rule for using CAB-Ke in Colombian valuation practice. Professionals must follow the rule appropriate for the sector's tier classification and provide the required disclosure (Table 12).

Tier	How to use CAB-Ke	Required actions in the report
Tier 1 — Strong Professional Evidence	CAB-Ke as principal beta. Damodaran shown for international comparability.	*Report 90% and 95% CIs; dated macro parameters; asyn disclosure.*
Tier 2 — Professional Use	CAB-Ke as principal professional beta with mandatory sensitivity.	*90% CI, Damodaran benchmark, Gap%, CRP sensitivity*
Tier 3 — Indicative	CAB-Ke as indicative beta with shrinkage; mandatory comparison against Damodaran.	*Comparison against Damodaran, BKS, sectoral median;*
Tier 4 — Fallback	Fallback blend (BKS 50% + median 30% + Damodaran 20%); only with judgment.	*Disclose insufficient local evidence; 3 components separ reason.*
Tier 5 — Special Treatment	Use separate methodology (traded bank comparables or other framework).	*State that CAB-Ke does not apply and reference the alter used.*

Independent of tier, the 30% Gap Rule applies: when |Gap| versus Damodaran exceeds 30%, the valuation report must include mandatory sensitivity with Damodaran benchmark and, where appropriate, a midpoint scenario.

B. Standard Disclosure Text by Tier

The following disclosure language is ready for insertion into professional valuation reports. Professionals must adapt it to the specific sector and firm being valued. See Appendix C for the full templates.

XII. LIMITATIONS

This section acknowledges substantial framework limitations. We organize them into ten categories.

(1) Annual data frequency. T = 19 imposes a structural ceiling on statistical power. The framework detects Hill-Stone betas of approximately 0.70 with $\alpha = 0.05$ and 80% power. Smaller effects are under-rejected. A quarterly extension would substantially improve power but is not implemented.

(2) Z-score look-ahead bias. Z-score parameters calibrated on data overlapping the analysis window by 37%. Section X.J reports robustness from the EMBI-based distress factor that eliminates this bias; main findings are preserved.

(3) Open-interval vs capped calibration. Spearman $\rho = 0.955$; nevertheless, 15 of 27 flagged at boundary and 8 of 27 with material change > 0.10. Open-interval is headline; capped is alternative.

(4) Strict Tier 1 count is small. Only one sector meets the strict symmetric dual-factor academic criterion. CAB-Ke's Tier 1 uses asymmetric criterion and contains three sectors. Both are reported transparently.

(5) Tier 4 Fallback count is large. Sixteen sectors require Fallback. Structural consequence of annual emerging-market data. Each Tier 4 sector has explicit Fallback Reason.

(6) Financial sector exclusion. CIIU 64, 66 permanently excluded. The professional must use a separate framework for financial intermediaries.

(7) Heterogeneity within CIIU 2-digit sectors. Each sector aggregates firms with different sub-activities. An extension to CIIU 3-digit would improve precision but is not implemented.

(8) Sensitivity to macro parameters. The Ke output is sensitive to CRP, ERP, Rf, π. The May 2026 snapshot must be updated to the valuation date.

(9) Fallback rule is not statistically optimal. The fallback blend is a transparent professional rule rather than a statistically optimal estimator. The quality-score weighted alternative remains as future work pending the availability of required inputs.

(10) Do not interpret all tiers as validation. Tier 1 is not strict academic validation; only one sector attains the latter. Tiers 2-4 explicitly ARE NOT validation. Professionals must respect this distinction in valuation reports.

XIII. CONCLUSION

This paper proposes the CAB-Ke framework, a tiered hierarchical accounting-beta procedure for estimating sector-level cost of equity for Colombian non-listed firms under emerging-market data constraints. The framework's central premise is that strict academic validation under conventional thresholds cannot be met for most sectors in a data-constrained emerging market, and the appropriate professional response is not to refuse to estimate or to produce point estimates without disclosure, but to construct a stratified reliability classification that distinguishes among strict academic, strong professional, and emerging-market indicative evidence standards, and assigns each sector to one of five reliability tiers.

The framework's primary outputs are the five-tier classification of the 27 Colombian non-financial sectors plus the 2 excluded financial sectors; the complete sector cost-of-equity matrix with sector-specific β_U from open-interval Vasicek shrinkage toward a CAPM-neutral prior, β_L under Hamada at the Colombian standard rate, USD Ke under Pereiro Adjusted Hybrid CAPM, COP Ke under Fisher conversion with long-run inflation expectations, 90% and 95% confidence intervals, the Damodaran benchmark, and required tier-specific disclosures. Three sectors qualify as Tier 1 Strong Professional Evidence (CIIU 6, 70, 77). Six as Tier 2 Professional Use (CIIU 9, 22, 25, 46, 47, 55). Two as Tier 3 Indicative (CIIU 17, 82). Sixteen require Tier 4 Fallback estimation. Two financial sectors are Tier 5 Special Treatment.

The framework's central substantive finding is rank orthogonality between locally estimated CAB-Ke accounting betas and the international Damodaran benchmark (Spearman $\rho \approx -0.20$). The orthogonality is interpretable: the Damodaran benchmark reflects average emerging-market sectoral composition and supports international comparability across transactions; the CAB-Ke beta reflects local Colombian accounting evidence and supports sectoral discrimination within Colombia. The two methodologies are complementary, addressing different professional needs. CAB-Ke does not seek to replace Damodaran. CAB-Ke contributes locally informed Colombian evidence; Damodaran remains the external comparability benchmark. The framework's matrix delivers a Ke estimate for every non-financial sector available in the Colombian SIREM panel, classified by reliability tier.

CAB-Ke does not eliminate uncertainty in Colombian private-company valuation. It makes uncertainty explicit, classifies it, and converts it into a professional decision rule for estimating sector-level cost of equity.

The framework's contribution is not statistical certainty, but disciplined professional estimation under uncertainty.

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Appendix A. Variable Definitions

This appendix provides formal definitions of all variables used in the CAB-Ke framework.

Variable	Definition
ROA_{s,t}	Asset-weighted operating return on assets in sector s, year t. Eq. (1).
ROA_{M,t}	Asset-weighted operating ROA across all firms in cohort, year t.
PD_{M,t}	Asset-weighted aggregate default probability, year t. Derived from firm-level Z-scores (eq. 2).
β_{HS,s}	Hill-Stone profitability beta. Slope of ΔROA_s on ΔROA_M (eq. 3).
β_{DA,s}	Distress-augmented beta. Slope of ΔROA_s on ΔPD_M (eq. 4), sign-inverted.
β_{BKS,s}	Beaver-Kettler-Scholes accounting variables beta (eq. 5).
β_{IV,s}	Inverse-variance combination of β_HS and β_DA (eqs. 6-7).
β_{GLS,s}	GLS combination considering residual correlation ρ≈0.45 (eq. 8).
β_{open,s}	Open-interval Vasicek posterior with CAPM-neutral prior β_0=1.0, σ_0=0.40 (eq. 9). PRIMARY calibration.
β_{capped,s}	clip(β_open,s, 0.40, 1.60). Comparability alternative specification.
β_{shrunk,s}	Bayesian posterior with tighter prior σ_0=0.30 for Tier 3 sectors.
β_{fallback,s}	Baseline blend: 0.50·β_BKS + 0.30·β_median + 0.20·β_Damodaran. Applied for Tier 4.
β_{CAB,s}^{final}	Final tier-specific unlevered beta: β_open (Tiers 1-2), β_shrunk (Tier 3), β_fallback (Tier 4).
β_{L,s}	Hamada-levered beta: β_U·[1+(1−τ)·D/E] (eq. 12).
β_{Damo,s}	Damodaran emerging-market unlevered beta (2026 update). External benchmark and fallback rule input.
Ke_{USD,s}	USD cost of equity under Pereiro Adjusted Hybrid CAPM: Rf_USD + CRP_COL + β_L·ERP_US (eq. 13).
Ke_{COP,s}	COP cost of equity under Fisher conversion with long-run inflation (eq. 14).
D/E_s	Sector median debt-to-equity ratio from the SIREM panel.
τ	Effective corporate tax rate. Headline τ = 35% (Ley 2277/2022). Sector-specific in sensitivity.
Rf_USD	U.S. 10-Year Treasury yield (Fed H.15).
ERP_US	U.S. equity risk premium from Damodaran (2026 update).
CRP_COL	Colombia country risk premium. Headline 2.85% (Damodaran); range to 3.30% (EMBI+ adjusted).
π_COL, π_US	Inflation in Colombia and U.S. Long-run (BanRep target, Fed expected) as headline.
Tier	Reliability tier from 1 to 5 (Table 4).
p_HS_BH, p_DA_BH	Benjamini-Hochberg-corrected p-values for HS and DA components.

Appendix C. Professional Disclosure Templates

This appendix provides ready-to-use disclosure language for Colombian valuation reports. Professionals must adapt the text to the specific sector and firm being valued. The structure of each disclosure block is: (i) tier classification statement; (ii) suitability for valuation use; (iii) required disclosures; (iv) explicit limitation acknowledgment where applicable.

Table 12 — Required Disclosure by Tier

The following templates are intended for direct inclusion in Colombian valuation reports using a CAB-Ke beta. Each template corresponds to one of the five reliability tiers.

Tier	Disclosure template (ready for insertion in valuation report)
Tier 1 — Strong Professional Evidence	*"The CAB-Ke beta is classified as Tier 1 — Strong Professional Evidence. It is suitable as principal valuation input under Colombia provided that the valuation report discloses the confidence interval, the Damodaran benchmark, and sensitivity to CRP, inflation, D/ assumptions. The Tier 1 criterion is asymmetric (BH p<0.05 on Hill-Stone AND p<0.10 on distress-augmented); it must not be interp symmetric dual-factor academic validation, which only one sector attains in the framework."*
Tier 2 — Professional Use	*"The CAB-Ke beta is classified as Tier 2 — Professional Use. It does not constitute strict academic validation but is suitable for prof use in Colombia with mandatory sensitivity analysis and benchmark disclosure. The report must include the 90% confidence interval emerging-market benchmark, the percentage gap relative to that benchmark, Ke sensitivity to alternative country-risk premium value 3.30%), and a narrative justifying the sectoral choice."*
Tier 3 — Indicative	*"The CAB-Ke beta is classified as Tier 3 — Indicative. The estimate is economically interpretable but must be used with shrinkage, b comparison, and explicit uncertainty disclosure. The Damodaran benchmark and BKS specification are reported as comparable alte analyst must provide explicit narrative justifying indicative use rather than reliance on Damodaran or BKS, and must not present thi validated."*
Tier 4 — Fallback	*"The CAB-Ke beta for this sector is classified as Tier 4 — Fallback. Local accounting evidence is insufficient for a reliable sector-sp estimate combines BKS accounting variables (50%), the cross-sectional median (30%), and the Damodaran benchmark (20%). The A the matrix documents the specific reason for this classification (typically sign inconsistency on Hill-Stone, implausibly low open-inte insufficient raw p-values). It must be used only with analyst judgment and explicit disclosure that local time-series evidence does not specific estimate."*
Tier 5 — Special Treatment	*"The CAB-Ke framework does not apply to this sector because SIREM's accounting debt-to-equity ratio does not capture customer-d relevant to financial-intermediary equity valuation. A separate beta methodology based on traded Colombian bank comparables or i financial-sector comparables with appropriate country-risk adjustment is required. The CAB-Ke matrix does not produce a Ke estim CIIU 66."*