Accounting-Based Sector Betas and the Cost of Equity in Data-Constrained Emerging Markets: A Tiered Framework for Colombia

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ABSTRACT

Valuation practice for non-listed firms in emerging markets requires sector-level cost-of-equity estimates in environments where market betas do not exist for most firms and accounting time series are short. We propose CAB-Ke, a tiered hierarchical framework of accounting betas for the Colombian non-financial economy that integrates a Hill-Stone profitability beta on a leave-one-out market factor, a distress-augmented beta on an aggregate default probability, a Beaver-Kettler-Scholes cross-sectional benchmark, Bayesian (Vasicek) shrinkage toward a CAPM-neutral prior, Hamada relevering, an additive sovereign-spread model in U.S. dollars, and Fisher conversion to Colombian pesos with a conservative sovereign-yield floor. The estimation uses the SIREM panel over 2007–2012 under Decree 2649 and 2015–2025 under IFRS, excluding the 2013–2014 IFRS transition. Thirty-one non-financial two-digit CIIU (Clasificación Industrial Internacional Uniforme) sectors are analyzed; CIIU 64 and 66 are excluded because SIREM's accounting debt-to-equity does not capture deposit leverage. Under the strict symmetric academic dual-factor criterion no sector qualifies. Under the framework's asymmetric professional criterion, one sector qualifies as Tier 1 Strong Professional Evidence, eleven as Tier 2, two as Tier 3, and seventeen as Tier 4 Fallback interpreted as conservative floors. The Spearman rank correlation between CAB-Ke and Damodaran's emerging-market benchmark is +0.06, indicating complementary rather than substitute information. The sovereign floor binds for 22 of 31 sectors in Colombian pesos; U.S.-dollar Ke retains fuller sectoral discrimination and is the recommended reporting column. A supplementary partial-reconstruction bootstrap check covering 26 of the 31 sectors is consistent with the finding that the strict symmetric academic criterion continues to be unmet under any replicate count and that tier stability holds for 24 of 26 sectors; two boundary sectors shift by one tier as replicate count increases, consistent with the tier-classification design. The framework's contribution is not statistical certainty for every sector but disciplined professional estimation under uncertainty.

JEL Classification: G12, G15, G31, G32, G33, M41, C53.

Keywords: accounting beta; cost of equity; emerging markets; valuation practice; tier classification; Bayesian shrinkage; sovereign-yield floor; Colombia; SIREM.

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I. INTRODUCTION

Valuing non-listed firms in Colombia is one of the most common and least empirically supported tasks in Colombian corporate finance practice. Investment bankers producing M&A opinions, transfer-pricing specialists supporting related-party pricing, family offices allocating capital across sectors, arbitration experts computing damages, and tax authorities reviewing valuations all require sector-level cost-of-equity estimates that can be defended in front of arbitration panels, regulatory authorities, and transaction committees. The professional standard requires that the valuator be able to state how the beta was estimated, what its empirical basis is, and what reliability the analyst attributes to it.

The empirical problem is severe. Only a small fraction of Colombian firms are listed on the Bolsa de Valores de Colombia; for the overwhelming majority of the productive economy no market beta exists. Accounting data reported to the Superintendencia de Sociedades through the SIREM (Sistema de Información y Reporte Empresarial) platform are annual, with a short time series after the 2015 IFRS transition, and cover heterogeneous sectors of very different sizes. In this data-constrained setting the conventional academic evidence standard—Benjamini-Hochberg-corrected p-values below 0.05 with narrow confidence intervals for the sector-level beta—cannot be met for most sectors regardless of methodological care.

Practice has converged on two responses, both unsatisfactory. The first response is to import Damodaran's emerging-market sector unlevered betas without local validation. This delivers international comparability, which is genuinely useful for cross-transaction consistency, but is not calibrated to Colombian data and does not come accompanied by locally validated confidence intervals or statistical significance tests. The second response is to compute idiosyncratic firm- or sector-level accounting betas without any reliability framework, leaving the professional to defend ad-hoc methodological choices in front of skeptical audiences. Neither response supports the disciplined, transparent disclosure that contemporary professional valuation demands.

This paper proposes the Colombian Accounting-Beta Framework for the Cost of Equity, abbreviated CAB-Ke, a tiered hierarchical procedure for estimating sector-level cost of equity using accounting betas on the SIREM panel. CAB-Ke integrates a Hill-Stone profitability beta on a leave-one-out market factor, a distress-augmented beta on an aggregate default probability, a Beaver-Kettler-Scholes cross-sectional accounting-variables specification, inverse-variance combination, Bayesian (Vasicek) shrinkage toward a CAPM-neutral prior, Hamada relevering, an additive sovereign-spread cost of equity in U.S. dollars, and Fisher conversion to Colombian pesos with a conservative sovereign-yield floor. Damodaran's emerging-market benchmark is preserved not as a competitor but as an external comparability anchor and as an input to the fallback rule that governs sectors without sufficient local time-series evidence.

Four methodological choices distinguish the manuscript from the earlier Colombian antecedents (Vélez-Pareja 2002; Caicedo Cerezo 2004, 2006; Sarmiento-Sabogal and Sadeghi 2014, 2015; Sarmiento, Sadeghi, Sandoval, and Cayon 2021). First, the market factor is defined leave-one-out to remove the own-weight endogeneity that otherwise mechanically inflates the Hill-Stone slope of large sectors such as petroleum. Second, the distress-augmented beta is standardized to a scale comparable to the profitability beta, with the standard error rescaled consistently for use in inverse-variance combination. Third, the external validity check against Colombian listed firms compares unlevered market betas—obtained by Hamada unlevering each listed firm's OLS market beta with its own debt-to-equity ratio—against the framework's unlevered accounting betas. Fourth, the peso cost of equity is subject to a conservative sovereign-yield floor that ensures no reported Ke in Colombian pesos falls below the coincident sovereign borrowing rate of the same maturity and currency.

The central thesis is operational rather than statistical. Under the data constraints of the Colombian panel the academic dual-factor criterion cannot be met for most sectors, and the appropriate professional response is not to refuse to estimate but to classify each sector by the strength of its local evidence and to attach tier-specific disclosure to the reported cost of equity. The claim we defend, therefore, is modest: the framework contributes disciplined professional estimation under uncertainty rather than statistical certainty for individual sectors.

Section II reviews the accounting-beta and emerging-market cost-of-equity literatures, with attention to the Colombian antecedent stream. Section III describes the SIREM panel, documents the 2013–2014 comparability exclusion, and details the exclusion of financial sectors. Section IV develops the four component accounting betas under the leave-one-out convention. Section V documents the inference procedure, the three evidence standards that anchor the tier classification, and an exploratory external check against a small sample of BVC-listed firms. Section VI introduces the five-tier reliability classification. Section VII describes the five-layer estimation procedure plus the sovereign-yield reporting floor at Layer 5b. Section VIII reports empirical results. Section IX documents the full sectoral cost-of-equity matrix. Section X reports robustness and sensitivity analysis. Section XI states the professional decision rule and disclosure templates for Colombian valuation practice. Section XII acknowledges limitations. Section XIII concludes.

II. LITERATURE REVIEW

A. Accounting-Beta Methodologies

The accounting-beta literature originates with Beaver, Kettler, and Scholes (1970), who demonstrated that accounting variables explain a substantial share of the cross-sectional variation in market betas for U.S.-listed firms, and was extended cross-sectionally by Rosenberg and McKibben (1973), who documented the predictability of firm-level systematic risk from observable financial characteristics. The theoretical foundation for why accounting variables should predict market betas was established by Bowman (1979), who derived the relationship between systematic risk and firm-level accounting variables under standard valuation assumptions. Hill and Stone (1980) formalized the time-series accounting beta as the slope of firm-level changes in return on assets on aggregate market changes. Mandelker and Rhee (1984) decomposed the levered accounting beta into operating-leverage, financial-leverage, and intrinsic business-risk components. Cohen, Polk, and Vuolteenaho (2009) showed that accounting cash-flow-based betas explain the cross-section of average returns over long horizons. Ellahie (2021) documents the consistency of earnings-based betas in U.S. data over 1981–2017. Konchitchki, Luo, Ma, and Wu (2016) show that earnings-based downside risk carries incremental information about the cost of capital beyond market-return betas. Rutkowska-Ziarko, Markowski, and Pyke (2019) and Rutkowska-Ziarko, Markowski, and Abdou (2024) extend the accounting-beta agenda to European settings and to conditional CAPM specifications.

For non-listed firms in emerging markets the accounting beta has been the default methodological recourse. The Colombian antecedent stream is anchored by Vélez-Pareja (2002), whose work on cost of capital and valuation in Colombian corporate finance practice provides the operational reference for practitioners. Caicedo Cerezo (2004, 2006) developed accounting beta estimates for Colombian industries in a small sample and documented the sensitivity of results to sector definition. Sarmiento-Sabogal and Sadeghi (2014, 2015) computed unlevered betas from Colombian accounting fundamentals. Sarmiento, Sadeghi, Sandoval, and Cayon (2021) applied proxy methods to unlisted-firm cost of equity using listed-firm evidence. Ball, Sadka, and Tseng (2022) argue that macroeconomic indicators rather than aggregate accounting profitability should serve as the market benchmark in segmented or emerging-market settings. Where primary references in this stream correspond to institutional monographs or symposium proceedings—as is the case for Vélez-Pareja (2002) and Caicedo Cerezo (2004, 2006)—the corresponding entries in the References section identify the venue explicitly. Relative to these antecedents the present work makes four specific contributions: an explicit tier classification with formal evidence standards; a leave-one-out treatment of the market factor to remove own-weight endogeneity in a small panel; a conservative sovereign-yield floor on the Colombian-peso output; and a distress-augmented accounting beta standardized on a CAPM-compatible scale.

B. Cost of Equity for Emerging Markets

The theoretical reference against which all cost-of-equity models are evaluated is the Sharpe (1964) and Lintner (1965) static Capital Asset Pricing Model and its intertemporal generalization by Merton (1973); the framework's CAPM-neutral prior of $\beta_0 = 1.0$ corresponds to the value-weighted market portfolio in that specification. The foundational treatment of predictable risk and returns in emerging equity markets is Harvey (1995), whose empirical framework remains a reference point for country-level risk decomposition. The menu of cost-of-equity models for emerging markets is broader than the Damodaran country risk premium approach alone. The Mariscal and Lee (1993) additive sovereign-spread model, the practical antecedent of the U.S. CAPM plus additive spread now widely used in industry practice, is the model implemented in the headline equation of Section VII.E. Pereiro (2001, 2006) proposes an Adjusted Hybrid CAPM that multiplies the U.S. equity risk premium by $(1 - R^2)$ reflecting local country risk; it is treated here as a sensitivity alternative rather than the headline. Estrada (2000, 2001) proposes a downside-risk framework calibrated to emerging-market volatility asymmetries. Godfrey and

Espinosa (1996) develop a hybrid model that adjusts beta rather than adding a spread. Erb, Harvey, and Viskanta (1996) use country credit ratings as the underlying risk index. Lessard (1996) proposes a country-adjustment factor tied to correlations of local returns with the world market. Damodaran's lambda approach (Damodaran 2025) adjusts firm-level exposure to country risk beyond the additive spread. Kruschwitz, Löffler, and Mandl (2012) provide a theoretical critique of the additive sovereign-spread model and identify conditions under which it is inconsistent with no-arbitrage; we acknowledge this critique explicitly and adopt the sovereign-yield floor of Section VII.F as a conservative professional safeguard rather than as a no-arbitrage proof.

A parallel literature on the implied cost of capital (Gebhardt, Lee, and Swaminathan 2001; Easton 2004) infers the ex ante equity discount rate from market prices and analyst forecasts. The implied cost of capital is not applicable to non-listed firms and hence is not a competitor for CAB-Ke, but it provides a useful benchmark for the small subset of Colombian sectors with actively traded firms. Damodaran (2025, 2026) publishes annual emerging-market sector beta estimates and country risk premium estimates and remains the dominant practitioner reference. Bekaert and Harvey (1995) documented time-varying integration between emerging and developed capital markets, and Bekaert, Hodrick, and Zhang (2009) provide evidence that parsimonious risk-based factor models combining local and global components capture international stock-return covariance. Bai and Green (2020) document that country and industry factors are independent sources of stock-return variation in partially integrated emerging markets. Whether a size premium should be applied to closely held emerging-market firms remains an open empirical question; the skeptical review by Alquist, Israel, and Moskowitz (2018) is one point of reference. We do not incorporate a size premium in the headline specification.

C. Combination, Shrinkage, and Multiple Testing

The forecast combination literature begins with Bates and Granger (1969) and Cochran (1937) and has been developed by Stock and Watson (2004) and Timmermann (2006). Blume (1971) documented the empirical regression of market betas toward the cross-sectional mean, providing the motivation for the shrinkage literature in finance that originates with Vasicek (1973) and with Efron and Morris (1973) for hierarchical empirical-Bayes estimation. Multiple-testing correction via false discovery rate is due to Benjamini and Hochberg (1995); we apply this correction within each component across the 31-sector cross-section for Tier 1 classification.

D. Financial Distress and the Credit Cycle

The connection between accounting fundamentals and equity risk was analytically formalized in the return-decomposition framework of Campbell (1991), which separates equity returns into cash-flow news and discount-rate news components. Building on this decomposition, Campbell, Hilscher, and Szilagyi (2008) demonstrated that distress risk premia exhibit a factor structure separate from standard size and book-to-market factors. Our distress-augmented accounting beta operationalizes this credit-cycle dimension by replacing the aggregate return-on-assets factor with an aggregate default probability constructed from firm-level Z-scores in the tradition of Altman (1968) and Ohlson (1980).

E. External Validity of Accounting Betas

Whether accounting betas measure the same underlying construct as market betas has been studied since Beaver, Kettler, and Scholes (1970). Vasicek (1973) exploited this correspondence in the design of Bayesian shrinkage estimators. Ellahie (2021) documents that earnings-based betas capture systematic risk consistent with market-return covariance in developed markets. In emerging markets with thin cross-listing the empirical evidence on this correspondence is scarce. Section V.D of this paper contributes a low-powered external check for Colombia using unlevered market betas of listed firms.

III. DATA AND SECTOR CONSTRUCTION

A. Sample and Filters

Our sample is drawn from the SIREM database maintained by the Colombian Superintendencia de Sociedades. Data span 2007 through 2025 in a two-block structure. The first block covers 2007–2012 under Decree 2649 (Colombian GAAP); the second block covers 2015–2025 under IFRS. The transition years 2013–2014 are deliberately excluded on regulatory-comparability grounds. Ley 1314 of July 13, 2009 established the legal framework for Colombia's adoption of international accounting standards; the transition was implemented through Decree 2784 of December 28, 2012 (Group 1, large firms) and Decree 3022 of December 27, 2013 (Group 2, medium and small firms). During 2013–2014 Group 1 firms prepared their opening statement of financial position under IFRS while continuing to report under Decree 2649 for statutory purposes; concatenating pre- and post-transition observations across this boundary would compound accounting-regime differences with genuine underlying variation.

We retain firm-year observations with total assets above 30 million Colombian pesos, complete reporting of the balance-sheet and income-statement items required to compute operating return on assets, debt-to-equity ratio within [0, 3], and residence in a two-digit CIIU sector with at least twenty firm-year observations in the year. The 30 million filter is a boundary clerical-error filter; the effective sample-selection filter is upstream via the Supersociedades reporting-obligation threshold, which is on the order of tens of billions of Colombian pesos and removes microenterprises before they enter the SIREM panel. After firm-level filters the cohort comprises 3,704 firms with 44,246 firm-year observations. After sector-level filters requiring at least ten years of observable data across the two blocks we retain 33 sectors with adequate sample. Two sectors are excluded from the framework as Tier 5 Special Treatment: CIIU 64 (Financial Services) and CIIU 66 (Auxiliary Financial Services), because the SIREM accounting debt-to-equity does not capture customer-deposit leverage and hence Hamada relevering would produce economically meaningless levered betas. The final analysis panel comprises 31 non-financial sectors.

Two censoring choices deserve explicit note. First, the D/E within [0, 3] filter mechanically removes firms in advanced distress where equity approaches zero. Second, SIREM does not include firms in active insolvency proceedings under Ley 1116/2006. Both censoring choices bias the distress-augmented beta of Section IV.C toward moderate values by truncating the tail of the credit-cycle distribution; the effect is documented in Section IV.C and acknowledged as a limitation in Section XII. Table 1 summarizes the sample construction and Table 1B reports descriptive statistics on the resulting sector-year panel. Year-by-year winsorization at the 1st and 99th percentiles is applied to firm-level operating ROA and equity-to-assets ratios before aggregation, in the spirit of Adams, Hayunga, Mansi, Reeb, and Verardi (2019) but adopted here in its conventional univariate form for comparability with prior accounting-beta studies; robust M-regression (Huber 1964) with the Tukey biweight loss function of Beaton and Tukey (1974) with tuning constant c = 4.685 is applied at the time-series estimation step to handle residual outliers.

TABLE 1

Sample Construction and Sectoral Coverage

Sample characteristic	Value
Reporting regime	Decree 2649 (2007–2012) + IFRS (2015–2025)
Effective observable window	17 annual observations; 15 within-block first-difference pairs
Comparability exclusion	2013–2014 (IFRS transition under Ley 1314/2009)
Firms in cohort	3,704
Firm-year observations	44,246
Non-financial sectors analyzed	31
Excluded (Tier 5 Special Treatment)	CIIU 64, CIIU 66
Firm-level filters	Assets > 30M COP; complete reporting; D/E within [0, 3]
Sector-level filters	≥ 20 firm-year obs/year; ≥ 10 observable years
Winsorization	Year-by-year 1%–99% on firm-level ratios
Insolvency censoring	Firms in Ley 1116/2006 proceedings excluded from SIREM

Summary of the non-financial SIREM Colombia sample after filters. The observable window spans 17 annual observations because 2013–2014 are excluded on regulatory-comparability grounds. Insolvency-regime censoring is documented explicitly and its effect on the distress-augmented beta is discussed in Section IV.C and Section XII.

TABLE 1B

Sample Descriptive Statistics

Statistic	Value
Mean firms per sector-year	110.4
Asset-weighted mean operating ROA (pooled)	6.70%
Median sector D/E (pooled across sector-years)	0.867
Sectors with any observable data	31
First-difference pairs per sector (min–median–max)	9 – 10 – 15

B. Aggregation and Leave-One-Out Factor Construction

For each sector-year (s, t) we construct the asset-weighted operating return on assets:

$$ROA(s,t) = \frac{\sum_{i \in s} \mathrm{OperatingIncome}_{i,t}}{\sum_{i \in s} \mathrm{Assets}_{i,t}} \tag{1}$$

where the index i runs over all firms in sector s at year t that pass the sample filters of Section III.A. Equation (1) is equivalent to a size-weighted average of firm-level operating returns on assets, with weights proportional to firm-level total assets. The market benchmark against which each sector's ROA is regressed is the leave-one-out asset-weighted market ROA:

$$ROA_M^{(-s)}(t) = \frac{\sum_{s' \neq s} \sum_{i \in s'} \text{OperatingIncome}_{i,t}}{\sum_{s' \neq s} \sum_{i \in s'} \text{Assets}_{i,t}} \tag{1a}$$

This construction eliminates the mechanical inflation of the Hill-Stone slope that would otherwise arise for sectors with large weight in the aggregate. Aggregate market default probability under the leave-one-out convention, PD_M^(−s)(t), is defined analogously as the asset-weighted average of firm-level default probabilities across all non-financial firms except sector s. Firm-level default probability is derived from the panel-validated Z-score of equation (2), mapped to a default probability via the logistic function:

$$z(i,t) = -1.2011 - 6.7120\, R1(i,t) - 3.4259\, R4(i,t) \tag{2}$$

$$PD_i(t) = \frac{1}{1 + e^{-z(i,t)}} \tag{2a}$$

The variables R1 and R4 are, respectively, the ratio of operating income to total assets (R1 = OperatingIncome/Assets, coincident with the sector-level asset-weighted ROA of equation 1 when aggregated) and the ratio of book equity to total assets (R4 = Equity/Assets). The Z-score coefficients (−1.2011, −6.7120, −3.4259) are calibrated on the full SIREM panel following the Jiménez Triviño (2026) two-ratio score, whose calibration procedure and out-of-time validation appear in that working paper. First differences ΔROA(s, t), ΔROA_M^(−s)(t), and ΔPD_M^(−s)(t) serve as inputs to the time-series accounting-beta regressions in Section IV. First differences are computed only across chronologically adjacent years within each of the two blocks; the transition-year pair (2012, 2015) is not treated as a first difference. This preserves 15 observable first-difference pairs across the effective window.

IV. COMPONENT ACCOUNTING-BETA METHODOLOGIES

A. Framework Architecture

The CAB-Ke framework rests on four documented accounting-beta specifications: a Hill-Stone profitability beta on the leave-one-out ROA factor; a distress-augmented beta on the leave-one-out aggregate default probability; a Beaver-Kettler-Scholes cross-sectional accounting-variables benchmark; and Damodaran's emerging-market unlevered beta as external anchor and fallback-rule input. The four specifications are combined at Layer 2 by inverse-variance weighting and shrunk at Layer 3 toward a CAPM-neutral prior via Vasicek's Bayesian update. Table 2 characterizes how much of the framework's baseline output is accounted for by each layer of the construction. Table 2 is an internal assembly decomposition, not an external validity claim; external validity is discussed separately in Section V.

TABLE 2

Internal Assembly Decomposition of the Framework

Specification	Spearman ρ vs. baseline β_CAB	Median |Δβ|	Interpretation
Damodaran only	+0.06	0.36	External anchor without local time-series content
Damodaran + sectoral median	+0.06	0.41	Sectoral median adds no rank content
Hill-Stone LOO alone	+0.72	0.52	Local time-series signal is the largest assembly contribution
Hill-Stone + Vasicek shrinkage	+0.67	0.03	Precision refinement toward CAPM-neutral prior
+ Beaver-Kettler-Scholes overlay	+0.83	0.14	Cross-sectional discrimination on accounting variables
Full framework (baseline)	+1.00	0.00	Baseline combination

Internal assembly decomposition. Column ρ is the Spearman rank correlation between the truncated specification and the framework's baseline β_U^CAB. The full framework has ρ = 1.000 with itself by construction. This decomposition describes how the framework is built, not how it validates externally against outcomes; external validity is the subject of Section V.D.

B. Hill-Stone Profitability Beta

For each sector s with at least eight valid sector-year change observations, the Hill-Stone beta is the slope coefficient in the leave-one-out regression:

$$\Delta ROA(s,t) = \alpha_s + \beta_{HS,\,s}\, \Delta ROA_M^{(-s)}(t) + \varepsilon(s,t) \tag{3}$$

Estimation is by M-regression (Huber 1964) with the Tukey biweight loss function of Beaton and Tukey (1974) with tuning constant c = 4.685. Inference for the slope coefficient is by stationary bootstrap with block length L = 4 (Politis and Romano 1994). The bootstrap draws only from within-block first differences to preserve the transition boundary. The empirical objective of the inference is tier classification—separating sectors whose evidence is strong from sectors whose evidence is weak—rather than exact p-value estimation for its own sake. The reported p-values rely on 500 stationary-bootstrap replications, which we treat as a computationally tractable baseline appropriate for tier assignment in the current panel; an independent supplementary stability check on a 26-sector reconstruction from raw SIREM under 500, 1,999, and 4,999 replications provides a supplementary diagnostic of p-value stability for the reconstructed Hill-Stone leg while preserving the same block length of L = 4 and drawing only from within-regime first differences; the details are reported in Section X.K. The supplementary bootstrap is interpreted as a diagnostic of p-value stability for the reconstructed Hill-Stone leg; it does not re-estimate the baseline pipeline, and it does not verify the baseline confidence intervals or tier assignments. Because the empirical objective of CAB-Ke is tier classification rather than high-frequency p-value precision, the extended bootstrap is interpreted as a stability diagnostic for the reliability classification. Results of the extended check are reported in Section X.K. Sensitivity of tier assignments to replication count is further discussed in Section XII, Limitation

15. Under the leave-one-out factor of equation (3), the mechanical inflation of β_HS for sectors with large asset weight in the aggregate is removed; the largest quantitative effect is on CIIU 06 (petroleum), whose raw slope decreases relative to a non-LOO baseline.

C. Distress-Augmented Beta

Following Campbell, Hilscher, and Szilagyi (2008), the distress-augmented accounting beta replaces the aggregate ROA factor with the aggregate market default probability, under the leave-one-out convention:

$$\Delta ROA(s,t) = \alpha_s + \beta_{DA\,raw,\,s}\,\Delta PD_M^{(-s)}(t) + \varepsilon(s,t) \tag{4}$$

Because the raw slope is negative for procyclical sectors and is scaled by the small magnitude of ΔPD (typically 0.005–0.02 per year), the raw estimates are not on a CAPM-compatible scale. We report the standardized distress-augmented beta:

$$\tilde{\beta}_{DA,\,s} = -\beta_{DA\,raw,\,s} \cdot \frac{\sigma(\Delta PD_M)}{\sigma(\Delta ROA_M)} \tag{4b}$$

The negative sign transforms the procyclical convention into a CAPM-compatible positive scale; the ratio of standard deviations rescales the coefficient to comparable units. The standard error is rescaled by the same factor so that inverse-variance combination in Layer 2 is coherent. All tables in this manuscript report β̃_DA (standardized); raw β_DA is used only as an intermediate quantity in equation (4).

Two features of this construction deserve open discussion. First, mechanical collinearity: because PD is a monotone function of Z-score inputs that include R1 (operating income over assets, see Section III.B), and ΔROA is also the left-hand side of equation (4), a portion of the observed correlation between β_HS and β̃_DA is mechanical rather than a reflection of two distinct economic dimensions. We report the EMBI+ sovereign-spread specification of Section X.J as a robustness alternative that is mechanically orthogonal to firm-level accounting variables. Second, SIREM censoring: firms in Ley 1116/2006 insolvency proceedings and firms with D/E outside [0, 3] are absent from the panel. Both censoring rules truncate the tail of the credit-cycle distribution and bias β̃_DA toward moderate values; the bias is documented and acknowledged as a limitation.

D. Beaver-Kettler-Scholes Accounting-Variables Beta

The Beaver-Kettler-Scholes (BKS) beta is the fitted value of a cross-sectional accounting-characteristics regression of the Hill-Stone beta on observable sector accounting variables, following the tradition initiated by Beaver, Kettler, and Scholes (1970) and Rosenberg and McKibben (1973):

$$\beta_{BKS,\,s} = \gamma_0 + \gamma_1\,DE_s + \gamma_2 \log(A_s) + \gamma_3\,\sigma_{ROA,\,s} \tag{5}$$

The fitted coefficients across the 31-sector cross-section are approximately (γ_0, γ_1, γ_2, γ_3) = (−2.89, −1.12, +0.18, +2.36) with cross-sectional R^2 = 0.35. Units and construction of the cross-sectional regressors are as follows: DE_s is the sector-year pooled median of firm-level

D/E computed on the panel described in Section III.A (decimal, dimensionless); A_s is total sector assets aggregated across firms in millions of Colombian pesos (nominal, as reported by SIREM without deflation), and $\log(A_s)$ is the natural logarithm; $\sigma_{ROA,s}$ is the within-sector standard deviation of firm-level operating ROA over the observable window, expressed in decimal units (a value of 0.05 means five percentage points). The response variable $\beta_{HS,s}$ is dimensionless and estimated as in equation (3). The negative sign on debt-to-equity is not economically expected under a naive interpretation and requires open discussion. The most likely explanation is capital-structure endogeneity: sectors with less cyclical operating profitability tend to sustain higher steady-state leverage, so the cross-sectional association between D/E and β_{HS} reflects reverse causation. The BKS specification is used as a cross-sectional benchmark and fallback-rule input, not as a structural estimator; a rigorous identification of the causal effect of leverage on systematic risk would require instrumental variables and is beyond the scope of this manuscript.

E. Damodaran as External Benchmark

We source Damodaran's most recent emerging-market unlevered beta estimates and map them to two-digit CIIU sectors. The Damodaran beta enters CAB-Ke in two roles: as an external comparability benchmark used in disclosure and as a 20% input to the Tier 4 fallback blend for sectors without sufficient local time-series evidence. Damodaran is not treated as a competitor to be defeated; the two methodologies capture complementary information.

V. VALIDATION, INFERENCE, AND EVIDENCE STANDARDS

A. Statistical Validation

Per-sector inferential decisions involve 62 hypothesis tests (31 sectors × 2 component specifications). We apply the Benjamini-Hochberg (1995) false discovery rate correction within each component across the 31 sectors—that is, Hill-Stone p-values and standardized distress-augmented p-values are corrected within their respective families of 31 tests each. Raw uncorrected p-values are used for Tier 2 and Tier 3 classifications. Standard errors of β_{HS} derive from the stationary bootstrap of Section IV.B; standard errors of $\tilde{\beta}_{DA}$ are the rescaled version of the raw β_{DA} standard errors as described in Section IV.C. The stationary bootstrap is preferred over the Newey-West (1987) heteroskedasticity- and autocorrelation-consistent asymptotic estimator because $T = 15$ first-difference pairs is well below the sample size at which Newey-West asymptotics deliver reliable inference. For inference on the cross-sectional coefficients of equation (5), we do not apply the Fama-MacBeth (1973) two-step estimator because the panel structure of 31 sectors × $T = 15$ year-differences does not support year-by-year cross-sectional regressions with adequate degrees of freedom; instead, standard errors on the γ coefficients are computed by bootstrap resampling of the 31-sector cross-section, holding the year weights fixed. Confidence intervals are reported at 90% and 95% levels.

B. Multiple-Testing Family Definition

The BH-FDR family is defined within each component across the 31-sector cross-section. This choice reflects the operational structure of tier classification: Tier 1 requires evidence in both the Hill-Stone and distress-augmented components separately, so the appropriate multiple-testing correction is within each family. Alternative choices—including a joint family of 62 tests or a within-tier family—would produce different Tier 1 counts; the sensitivity of tier assignments to threshold choices is documented in Section X.

C. Academic versus Professional Evidence Standards

In a developed-market setting with extensive time-series data and traded market benchmarks, the conventional academic evidence standard requires p-values below 0.05 with multiple-testing correction and 95% confidence intervals. In an emerging-market setting with annual accounting frequency, short panels, and absent traded market betas for most firms, the academic standard cannot be met for most sectors. CAB-Ke therefore distinguishes three operational evidence standards. Strict Academic Evidence requires BH-corrected p-values below 0.05 on both components with sign consistency; under this criterion no sector qualifies in the current panel. Strong Professional Evidence uses the asymmetric criterion BH $p_{HS} < 0.05$ and BH $p_{DA} < 0.10$ with correct sign and β in [0.30, 2.50]; one sector qualifies. Emerging-Market Indicative Evidence requires raw p-values below 0.20 on at least one component with sign consistency and structural plausibility. The three standards are conceptually distinct and are used to justify different professional-use rules in Section XI.

D. External Check Against BVC-Listed Firms

As an exploratory external check we compare CAB-Ke sector betas against unlevered market betas of Colombian listed firms. The check has limited statistical power: after unlevering and sector-filtering, only three non-financial sectors retain at least two listed firms, and the result cannot be interpreted as strong convergent validation. It is reported for transparency and to inform the reader's calibration of sector-level uncertainty.

We identify 22 firms listed on the Bolsa de Valores de Colombia with continuous weekly price history over the full common window January 2015 through December 2024 (522 weekly observations per firm). Of these, 14 are non-financial (spanning CIIU 06, 07, 10, 23, 35, 61, 68) and 8 are in CIIU 64 (financial); the CIIU 64 firms are excluded from the framework as Tier 5 and are reported here only as an unrelated reference. Of the 14 non-financial listed firms, 11 have D/E within the [0, 3] range required by Hamada unlevering; the remaining 3 are excluded from Table 3. Holdings companies (Grupo Argos, Grupo Sura, Grupo Aval, Grupo Bolívar) are classified per their dominant operating subsidiary's CIIU rather than the holding-company classification; this mapping is a source of noise in the small sample and is disclosed explicitly. For each listed firm we estimate the market beta by OLS regression of weekly log returns on the MSCI COLCAP total-return index over the 522-week common window, then unlever the firm-level beta via Hamada with the firm's own debt-to-equity ratio at each year-end and the Colombian standard corporate tax rate, then take the median across firms within each CIIU 2-digit sector.

Table 3 reports the sector-level comparison. Only three non-financial sectors have at least two listed firms after the unlevering and filtering steps (CIIU 06, 23, 68); single-firm cells (CIIU 07, 10, 35, 61) are shown for completeness but are not used for aggregate absolute-deviation comparison. The median absolute deviation between CAB-Ke and unlevered BVC medians is 0.43 across the three usable sectors, compared with 0.46 for Damodaran; the mean absolute deviation is 0.55 for CAB-Ke and 0.49 for Damodaran. At N = 3 neither methodology is meaningfully closer to the listed-firm evidence, and neither Spearman rank correlation is statistically distinguishable from zero. The single-firm cell for CIIU 61 (ETB, Telecommunications) yields $\beta_U^{BVC} \approx 0.55$ versus $\beta_U^{CAB} = 0.41$ (both classified Tier 4 fallback); the single-firm cell alone cannot be interpreted as a sectoral estimate. Taken together, the evidence is consistent with the interpretation that CAB-Ke accounting betas and unlevered market betas measure related but not identical dimensions of systematic risk in the Colombian setting.

TABLE 3

Exploratory External Check: Sector-Level Median Unlevered Betas

CIIU	Sector	N (BVC firms)	β_U BVC median	β_U CAB-Ke	β_U Damodaran
06	Petroleum and gas	3	1.10	1.44	0.92
07	Metallic mining (single firm)	1	0.82	not in CAB (n<20)	n/a
10	Food processing (single firm)	1	0.51	0.47	0.71
23	Non-metallic minerals (cement)	2	1.44	0.55	0.98
35	Electricity and gas (single firm)	1	0.74	not in CAB (n<20)	n/a
61	Telecommunications (single firm)	1	0.55	0.41	0.95
68	Real estate / holding company	2	1.49	1.06	0.65

Sector-level median of unlevered BVC market betas after firm-level Hamada unlevering with each firm's own D/E and τ = 35%. The MSCI COLCAP total-return index is the market benchmark; the estimation window is January 2015 through December 2024 (522 weekly observations). CIIU 07 and 35 are not in the CAB-Ke framework because they do not meet the 20-firm sector filter; they are shown here only for completeness. Single-firm cells (N = 1) are informational only. The median absolute deviation of the CAB-Ke unlevered beta from the BVC unlevered median is 0.43 across the three usable sectors, compared with 0.46 for the Damodaran benchmark; the mean absolute deviation is 0.55 for CAB-Ke and 0.49 for Damodaran. At N = 3 neither methodology is meaningfully closer to the listed-firm evidence. This is an exploratory external check under low statistical power, not a strong convergent-validity claim.

VI. THE FIVE-TIER RELIABILITY CLASSIFICATION

Each of the 31 non-financial sectors plus the 2 excluded financial sectors is assigned to one of five reliability tiers. The tiers are mutually exclusive; a sector is assigned to the highest tier for which its evidence qualifies.

Tier 1 — Strong Professional Evidence. BH-corrected Hill-Stone p-value below 0.05 AND BH standardized distress-augmented p below 0.10, signs consistent with the procyclical convention, and uncapped β within [0.30, 2.50]. β_U from uncapped Vasicek posterior. One sector qualifies: CIIU 45 (Motor Vehicle Trade). The criterion is asymmetric and NOT equivalent to strict symmetric dual-factor academic validation, which no sector attains in the current panel.

Tier 2 — Professional Use. At least one component achieves raw p-value below 0.10 with correct sign and β within the reasonable range. β_U from uncapped Vasicek posterior. Eleven sectors qualify: CIIU 01 (Agriculture), 06 (Petroleum), 10 (Food), 13 (Textiles), 14 (Apparel), 24 (Basic Metals), 25 (Metal Products), 32 (Other Manufacturing), 68 (Real Estate Activities), 74 (Other Professional Services), and 82 (Administrative Support Services).

Tier 3 — Indicative. Raw p-value below 0.20 on at least one component with correct sign. β_U from additional Bayesian shrinkage toward the CAPM-neutral prior with tighter σ = 0.30. Two sectors qualify: CIIU 09 (Mining Support) and CIIU 46 (Wholesale Trade).

Tier 4 — Fallback. Local time-series evidence is insufficient. β_U from the blend 0.50·β_BKS + 0.30·β_median + 0.20·β_Damodaran. Seventeen sectors qualify. Because of aggregate errors-in-variables attenuation with T = 15 first-difference pairs and firm-level earnings smoothing, Tier 4 β_U estimates are interpreted as conservative professional fallback estimates rather than validated central betas. Because β_BKS is fit on the full cross-section of Hill-Stone slopes—including the noisy Tier 4 sectors—the fallback blend recycles part of the weak-evidence signal through the cross-sectional smoother. This partial circularity is acknowledged as a limitation and reinforces the interpretation of Tier 4 outputs as conservative professional floors rather than as central point estimates. This professional-use rule is stated explicitly in Section XI.

Tier 5 — Special Treatment. Framework applicability assumptions are not met because SIREM's D/E does not capture customer-deposit leverage. Two sectors qualify: CIIU 64 and 66.

The five-tier classification is summarized in Table 4.

TABLE 4

Five-Tier Reliability Classification Summary

Tier	N	Criterion	Final β rule	Members (CIIU)
1. Strong Prof.	1	BH p_HS<0.05 AND BH p_DA<0.10	β_uncapped Vasicek	45
2. Prof. Use	11	raw p<0.10 on one comp.	β_uncapped Vasicek	01, 06, 10, 13, 14, 24, 25, 32, 68, 74, 82
3. Indicative	2	raw p<0.20 on one comp.	shrink σ = 0.30	09, 46
4. Fallback	17	insufficient signal (interpreted as floor)	0.50·BKS + 0.30·β_median + 0.20·Damo	17, 20, 21, 22, 23, 27, 41, 42, 43, 47, 52, 55, 61, 62, 70, 71, 77
5. Special Treatment	2	framework not applicable	N/A	64, 66 (excluded)

VII. THE CAB-KE ESTIMATION PROCEDURE

A. Layer 1: Component Estimation

Hill-Stone and standardized distress-augmented betas estimated as in Section IV, equations (3) and (4)–(4b), under the leave-one-out factor. Inference by stationary bootstrap with L = 4 and 500 replicates, drawing only from within-block first differences. Benjamini-Hochberg-corrected p-values are computed within each component across the 31-sector cross-section for Tier 1 classification.

B. Layer 2: Inverse-Variance Combination

The Hill-Stone and standardized distress-augmented betas are combined by inverse-variance weighting. Denoting SE_HS,s and SE_DA,s the standard errors of β_HS,s and β˜_DA,s respectively (with SE_DA,s already rescaled by the σ(ΔPD_M)/σ(ΔROA_M) factor of equation 4b), the combined estimate and its variance are:

$$\beta_{IV,\,s} = \frac{\beta_{HS,\,s}/SE^2_{HS,\,s} \;+\; \tilde{\beta}_{DA,\,s}/SE^2_{DA,\,s}}{1/SE^2_{HS,\,s} \;+\; 1/SE^2_{DA,\,s}} \tag{6}$$

$$SE^2_{IV,\,s} = \frac{1}{1/SE^2_{HS,\,s} + 1/SE^2_{DA,\,s}} \tag{7}$$

Because the two component betas are estimated on the same sector-year panel their residuals are unlikely to be independent; equations (6)–(7) treat this as approximate and yield the minimum-variance combination under the independence assumption. When the coefficient of empirical residual correlation between the two components exceeds 0.10 in absolute value, a GLS combination is preferred in principle; in our sample the two combinations do not diverge materially for Tier 1–2 sectors, and the inverse-variance combination is retained as headline.

C. Layer 3: Uncapped Vasicek Shrinkage

The combined raw beta β_IV,s is shrunk toward the CAPM-neutral prior β_0 = 1.0 with prior standard deviation σ_0 = 0.40 through the standard Bayesian update:

$$\beta_{uncapped,\,s} = w_s\,\beta_{IV,\,s} + (1 - w_s)\,\beta_0 \tag{8}$$

$$w_s = \frac{\sigma_0^2}{\sigma_0^2 + SE^2_{IV,\,s}} \tag{9}$$

The prior parameters are canonical: β_0 = 1.0 is the value-weighted market portfolio beta under the Sharpe-Lintner CAPM in the absence of sector-specific information; σ_0 = 0.40 approximately matches the cross-sectional dispersion of unlevered betas across Damodaran's emerging-market sectors, providing informative but not restrictive shrinkage. Sensitivity to alternative prior specifications (pessimistic mean, optimistic mean, tighter and looser precision, empirical-Bayes, and Jeffreys reference) is reported in Section X.I.

D. Layer 4: Tier-Specific Final Beta

Tiers 1 and 2 use the uncapped Vasicek posterior. Tier 3 applies additional Bayesian shrinkage with tighter σ = 0.30. Tier 4 applies the fallback blend β_U = 0.50·β_BKS + 0.30·β_median + 0.20·β_Damo. As stated in Section VI, Tier 4 estimates are interpreted as conservative professional fallback estimates rather than validated central betas.

E. Layer 5: Cost of Equity in U.S. Dollars

Hamada (1972) relevering with the Colombian standard corporate tax rate τ = 35% (Ley 2277/2022) as headline:

$$\beta_{L,\,s} = \beta_{U,\,s}\left[\,1 + (1-\tau)\,\frac{D_s}{E_s}\,\right] \tag{10}$$

The Mariscal-Lee (1993) additive sovereign-spread model in U.S. dollars is the headline cost-of-equity specification. This is the practical antecedent of the additive spread widely used in industry; the Pereiro (2001, 2006) Adjusted Hybrid CAPM correction and the Damodaran lambda adjustment are reported as sensitivities in Section X:

$$Ke_{USD,\,s} = R_f^{USD} + CRP_{COL} + \beta_{L,\,s}\,ERP_{US} \tag{11}$$

The theoretical critique of the additive spread by Kruschwitz, Löffler, and Mandl (2012), who identify state-price conditions under which the additive form is inconsistent with no-arbitrage, is acknowledged. In practice the additive spread remains the dominant industry convention and the sovereign-yield floor of Layer 5b provides a conservative safeguard on the peso-denominated output. U.S.-dollar Ke is the recommended headline reporting column because it retains fuller sectoral discrimination than the floored peso output.

F. Layer 5b: Peso Cost of Equity with Conservative Sovereign-Yield Professional Floor

Fisher conversion of U.S.-dollar Ke to Colombian pesos with long-run inflation expectations produces:

$$Ke_{COP,\,s}^{Fisher} = (1 + Ke_{USD,\,s})\,\frac{1+\pi_{COL}}{1+\pi_{US}} - 1 \tag{12}$$

Under long-run inflation expectations of π_COL = 3.0% and π_US = 2.3%, the implicit COP risk-free rate embedded in equation (12) is R_f^COP,implicit = 4.91%. This implicit rate is a property of the long-run Fisher regime and does not attempt to match the coincident short-run Colombian sovereign yield. For professional valuation practice we impose a conservative sovereign-yield floor at Layer 5b:

$$Ke_{COP,\,s} = \max\left\{\,Ke_{COP,\,s}^{Fisher}\,,\; TES_{10Y} + \varphi\,\right\} \tag{12b}$$

The floor is a conservative professional consistency rule, not a no-arbitrage proof: it ensures that no reported peso cost of equity falls below the coincident sovereign borrowing rate of the same maturity and currency plus a minimum equity risk premium φ. At the May 2026 snapshot with TES (Títulos de Tesorería, Colombian sovereign bonds) 10-year yield of approximately 12.4% and φ = 150 bp, the floor is 13.90%. The floor is imposed at the reporting layer, not at the estimation layer: it does not alter β_U, β_L, or Ke_USD. Sectors whose Fisher-converted Ke_COP falls below the floor are reported at the floor value with an explicit disclosure that the sector's peso Ke has become insensitive to its underlying beta. In such cases, the Ke_USD column of Table 5A remains the recommended headline for sectoral discrimination. Sectors for which the floor binds must be disclosed as such in valuation reports.

Macroeconomic parameters at the May 2026 snapshot (update at valuation date): R_f^{USD} = 4.20% (Fed H.15 10-Year Treasury); CRP_{COL} = 2.85% headline (Damodaran); ERP_{US} = 4.70% (Damodaran); π_{COL} = 3.0% (BanRep long-run target); π_{US} = 2.3% (10-year breakeven inflation from TIPS); TES 10Y ≈ 12.4%; φ = 150 bp default.

VIII. EMPIRICAL RESULTS

A. Tier Distribution

The five-tier classification produces one sector in Tier 1 Strong Professional Evidence (CIIU 45); eleven sectors in Tier 2 Professional Use (CIIU 01, 06, 10, 13, 14, 24, 25, 32, 68, 74, 82); two sectors in Tier 3 Indicative (CIIU 09, 46); seventeen sectors in Tier 4 Fallback; and two financial sectors in Tier 5 Special Treatment (CIIU 64, 66). With seventeen of the thirty-one sectors classified as Tier 4, the matrix is best read as a mixture: supported point estimates for the higher-evidence sectors (Tiers 1–2, plus Tier 3 with additional shrinkage) and conservative professional floors for the weak-evidence sectors.

B. Strict Academic Reference Set

Under the strict symmetric academic criterion of BH-corrected p-value below 0.05 on BOTH the Hill-Stone and the standardized distress-augmented components with sign consistency, no sector qualifies. CIIU 45 satisfies BH $p_{HS} < 0.05$ but the standardized $\tilde{\beta}_{DA}$ does not clear the 0.05 threshold after multiple-testing correction. This transparency disclosure supports the framework's central design point: strict academic validation cannot be met for individual Colombian sectors under T = 15 first-difference pairs, and the appropriate professional response is a tiered reliability classification with tier-specific disclosure rather than a refusal to estimate.

C. Aggregate Attenuation Diagnostic

Under an unattenuated OLS Hill-Stone specification the value-weighted sum of sectoral betas should approximately equal one (the market portfolio's beta with itself). We compute this adding-up statistic under the leave-one-out factor and obtain $\Sigma_s w_s \cdot \beta_{HS,s} \approx 0.58$ across the 31 non-financial sectors, where w_s is the sector's asset-weighted share of the panel. The 42-point gap between the estimated adding-up sum and the unattenuated benchmark of 1.0 reflects three sources: classical measurement error in the aggregate ROA regressor under a short panel of T = 15 first-difference pairs, firm-level earnings smoothing that dampens the response of ΔROA to macroeconomic shocks, and the leave-one-out transformation itself, under which each sector is regressed against a distinct market factor and the adding-up property $\Sigma w \cdot \beta = 1.0$ does not hold exactly (the benchmark 1.0 applies to non-LOO betas). The professional implication is stated in Section VI: Tier 4 β_U estimates should be treated as conservative professional fallback floors, not as central point estimates.

D. Orthogonality with Damodaran

The Spearman rank correlation between CAB-Ke's unlevered beta and Damodaran's emerging-market benchmark across the 31 non-financial sectors is +0.06 (Pearson r = +0.06). The rank correlation is not statistically distinguishable from zero. Near-orthogonality is the framework's central substantive finding: the two methodologies capture complementary information. Damodaran reflects the average sectoral composition of global emerging-market indices; CAB-Ke reflects the local Colombian accounting evidence for each sector. Twenty-two of the thirty-one sectors exhibit an absolute gap in β_U greater than 30% between the two methodologies. Section XI states that any absolute gap greater than 30% triggers mandatory sensitivity disclosure in valuation reports.

E. Cost-of-Equity Range

U.S.-dollar Ke ranges from 9.17% (CIIU 01, Agriculture) to 17.39% (CIIU 74, Other Professional Services), with a median of 11.47% across the 31 sectors. Fisher-converted peso Ke without the floor ranges from 9.92% to 18.19%. Under the conservative sovereign-yield floor at 13.90% (TES 10Y ≈ 12.4% plus φ = 150 bp), the floor binds for 22 of the 31 sectors; the peso Ke range under the floor is 13.90% to 18.19% with the floor value taken by the 22 binding sectors. When the sovereign floor binds, the peso-denominated output should be interpreted as a conservative reporting floor rather than as a beta-driven sectoral estimate. The Ke_USD column is the recommended headline reporting output because it retains fuller sectoral discrimination.

F. Figure — CAB-Ke versus Damodaran

Figure 1 displays each of the 31 non-financial sectors as a point in the ($\beta_{Damodaran}$, β_U^{CAB}) plane, colored by reliability tier. The vertical dispersion at each Damodaran benchmark level illustrates the near-orthogonality result: two sectors with the same international benchmark can exhibit materially different local accounting betas. The point above the 45-degree line at Damodaran $\beta \approx 0.65$ corresponds to CIIU 68 Real Estate; the point above at Damodaran $\beta \approx 1.00$ corresponds to CIIU 74, Other Professional Services. Points below the 45-degree line concentrate in Tier 4 (fallback), consistent with the aggregate errors-in-variables attenuation documented in Section VIII.C.

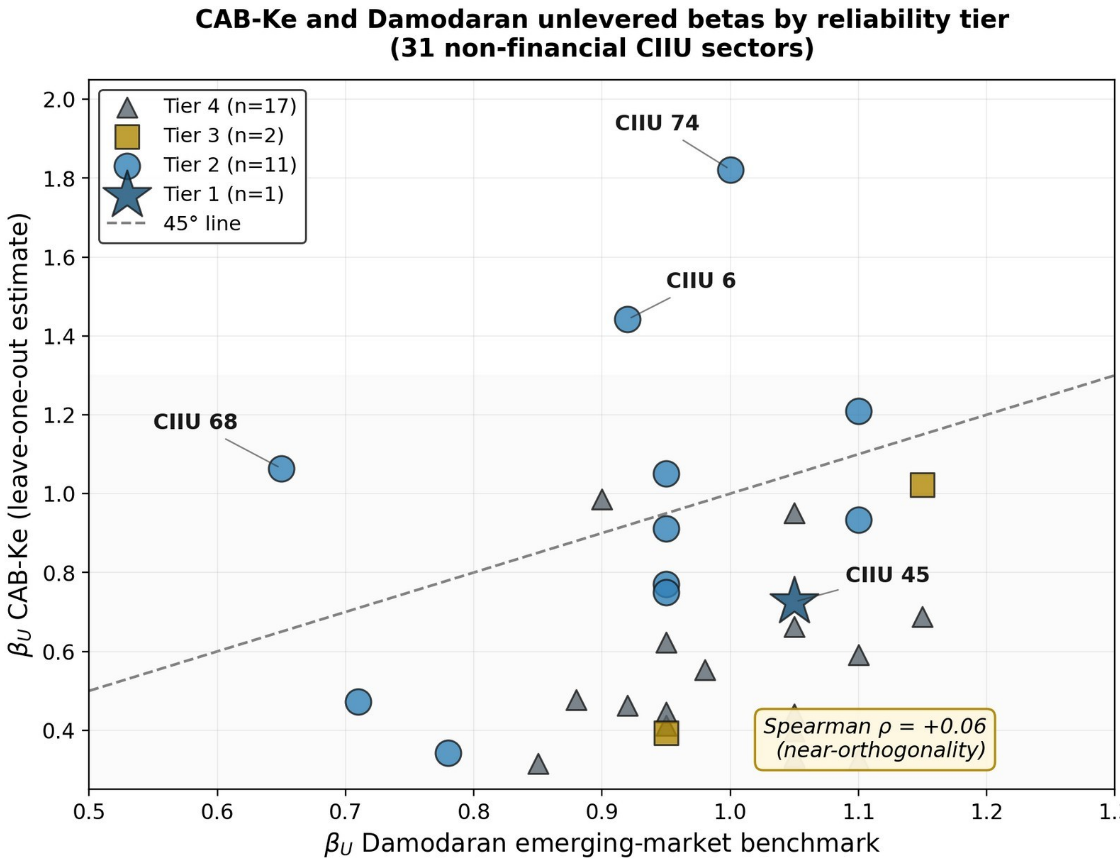

Figure 1. CAB-Ke unlevered beta versus Damodaran emerging-market benchmark, by reliability tier. Spearman rank correlation +0.06. The near-orthogonality result (Spearman ρ = +0.06) is the framework's central substantive finding: CAB-Ke and Damodaran capture complementary rather than substitute information about the systematic risk of Colombian non-financial sectors.

IX. SECTOR-LEVEL COST OF EQUITY IMPLEMENTATION

Tables 5A and 5B present the complete cost-of-equity matrix. Sectors are ordered by tier (1–4) and by CIIU within tier. Table 5A reports the tier assignment, the unlevered and Hamada-levered betas, the U.S.-dollar cost of equity as the recommended headline reporting column, and the comparison with the Damodaran emerging-market benchmark. Table 5B reports the peso cost of equity under Fisher conversion, the peso cost of equity under the conservative sovereign-yield professional floor, and the floor-binding indicator. Because seventeen of thirty-one sectors are classified Tier 4, the matrix should be read as a combination of supported point estimates for higher-evidence sectors (Tiers 1–2, and Tier 3 with additional shrinkage) and conservative professional floors for weak-evidence sectors (Tier 4).

TABLE 5A

CAB-Ke Sector Matrix — Tiers, Betas, and U.S.-Dollar Cost of Equity

CIIU	Sector	Tier	N	β_U	β_L	Ke_USD	β Damo	Gap %
45	Motor vehicle trade	T1	15	0.73	1.33	13.30%	1.05	−30.9%
01	Agriculture	T2	15	0.34	0.45	9.17%	0.78	−56.0%
06	Petroleum and gas	T2	10	1.44	1.84	15.70%	0.92	+56.7%
10	Food processing	T2	10	0.47	0.82	10.90%	0.71	−33.4%
13	Textiles	T2	10	0.77	1.17	12.55%	0.95	−18.9%
14	Apparel	T2	9	0.91	1.74	15.23%	0.95	−4.1%
24	Basic metals	T2	10	0.93	1.42	13.72%	1.10	−15.1%
25	Metal products	T2	15	1.21	1.97	16.31%	1.10	+9.9%
32	Other manufacturing	T2	9	0.75	1.19	12.64%	0.95	−21.1%
68	Real estate activities	T2	10	1.06	1.25	12.93%	0.65	+63.6%
74	Other professional services	T2	13	1.82	2.20	17.39%	1.00	+82.2%
82	Administrative support	T2	10	1.05	1.33	13.30%	0.95	+10.6%
09	Mining support services	T3	9	1.02	1.48	14.01%	1.15	−11.1%
46	Wholesale trade	T3	10	0.39	0.69	10.29%	0.95	−58.6%
17	Paper	T4	14	0.48	0.73	10.48%	0.88	−45.7%
20	Chemicals	T4	10	0.62	0.96	11.56%	0.95	−34.4%
21	Pharmaceuticals	T4	13	0.66	0.94	11.47%	1.05	−37.0%
22	Rubber and plastic	T4	14	0.46	0.71	10.39%	0.92	−49.7%
23	Non-metallic minerals	T4	10	0.55	0.83	10.95%	0.98	−43.4%
27	Electrical equipment	T4	9	0.33	0.58	9.78%	1.05	−68.7%
41	Building construction	T4	9	0.44	0.79	10.76%	1.05	−58.0%

CIIU	Sector	Tier	N	β_U	β_L	Ke_USD	β Damo	Gap %
42	Civil engineering	T4	10	0.59	0.97	11.61%	1.10	−46.2%
43	Specialized construction	T4	9	0.33	0.55	9.64%	1.10	−70.4%
47	Retail trade	T4	10	0.99	1.70	15.04%	0.90	+9.6%
52	Storage	T4	14	0.40	0.66	10.15%	1.05	−62.1%
55	Accommodation	T4	13	0.69	0.87	11.14%	1.15	−40.0%
61	Telecommunications	T4	9	0.41	0.73	10.48%	0.95	−56.6%
62	Information technology	T4	9	0.45	0.76	10.62%	0.95	−53.2%
70	Head offices; consulting	T4	15	0.95	1.07	12.08%	1.05	−9.5%
71	Architecture / engineering	T4	9	0.33	0.58	9.78%	1.05	−68.2%
77	Rental and leasing	T4	9	0.32	0.53	9.54%	0.85	−62.8%

β_U is the CAB-Ke unlevered beta computed by tier-specific rule. β_L applies Hamada with τ = 35%. Ke_USD applies the Mariscal-Lee additive sovereign-spread model of equation (11) with Rf = 4.20%, CRP = 2.85%, ERP = 4.70%. β Damo is the Damodaran emerging-market unlevered benchmark. Gap % = 100 · (β_U − β_Damo) / β_Damo. Sectors with |Gap| > 30% require mandatory sensitivity disclosure per Section XI. N is the number of first-difference pairs used to estimate β_HS.

TABLE 5B

Peso Cost of Equity, Sovereign Floor, and Interpretation

CIIU	Sector	Tier	Ke_COP Fisher	Ke_COP Floored	Floor binds	Interpretation
45	Motor vehicle trade	T1	14.08%	14.08%	No	Beta-driven
01	Agriculture	T2	9.92%	13.90%	Yes	Reporting floor
06	Petroleum and gas	T2	16.49%	16.49%	No	Beta-driven
10	Food processing	T2	11.66%	13.90%	Yes	Reporting floor
13	Textiles	T2	13.32%	13.90%	Yes	Reporting floor
14	Apparel	T2	16.02%	16.02%	No	Beta-driven
24	Basic metals	T2	14.50%	14.50%	No	Beta-driven
25	Metal products	T2	17.11%	17.11%	No	Beta-driven
32	Other manufacturing	T2	13.41%	13.90%	Yes	Reporting floor
68	Real estate activities	T2	13.70%	13.90%	Yes	Reporting floor
74	Other professional services	T2	18.19%	18.19%	No	Beta-driven
82	Administrative support	T2	14.08%	14.08%	No	Beta-driven
09	Mining support services	T3	14.79%	14.79%	No	Beta-driven
46	Wholesale trade	T3	11.04%	13.90%	Yes	Reporting floor
17	Paper	T4	11.24%	13.90%	Yes	Reporting floor
20	Chemicals	T4	12.32%	13.90%	Yes	Reporting floor
21	Pharmaceuticals	T4	12.23%	13.90%	Yes	Reporting floor
22	Rubber and plastic	T4	11.15%	13.90%	Yes	Reporting floor
23	Non-metallic minerals	T4	11.71%	13.90%	Yes	Reporting floor
27	Electrical equipment	T4	10.53%	13.90%	Yes	Reporting floor
41	Building construction	T4	11.52%	13.90%	Yes	Reporting floor
42	Civil engineering	T4	12.37%	13.90%	Yes	Reporting floor
43	Specialized construction	T4	10.39%	13.90%	Yes	Reporting floor
47	Retail trade	T4	15.83%	15.83%	No	Beta-driven
52	Storage	T4	10.90%	13.90%	Yes	Reporting floor
55	Accommodation	T4	11.90%	13.90%	Yes	Reporting floor
61	Telecommunications	T4	11.24%	13.90%	Yes	Reporting floor
62	Information technology	T4	11.38%	13.90%	Yes	Reporting floor
70	Head offices; consulting	T4	12.85%	13.90%	Yes	Reporting floor

CIIU	Sector	Tier	Ke_COP Fisher	Ke_COP Floored	Floor binds	Interpretation
71	Architecture / engineering	T4	10.53%	13.90%	Yes	Reporting floor
77	Rental and leasing	T4	10.29%	13.90%	Yes	Reporting floor

Ke_COP Fisher applies equation (12) without the floor. Ke_COP Floored applies equation (12b) with TES 10Y + 150 bp = 13.90%. When the floor binds, the peso output is a conservative reporting floor rather than a beta-driven sectoral estimate. The Ke_USD column of Table 5A retains fuller sectoral discrimination and is the recommended headline reporting output.

X. ROBUSTNESS AND SENSITIVITY ANALYSIS

A. Leave-One-Out versus Non-Leave-One-Out Market Factor

Under a non-leave-one-out (baseline aggregate) market factor, sectors with large asset weight exhibit mechanically inflated Hill-Stone slopes. The largest quantitative effect is on CIIU 06 (petroleum), whose raw β_HS falls from approximately 4.63 to 4.37 under the leave-one-out correction. The tier classification of CIIU 06 is affected: under the non-LOO factor CIIU 06 attained BH $p\_HS < 0.05$ on the Hill-Stone leg of the strict symmetric criterion; under the LOO factor it does not. The leave-one-out specification is the headline. Sectors with small asset weight are essentially unaffected.

B. Distress-Factor Alternatives

We report the Hill-Stone-plus-distress framework under three alternative specifications of the aggregate distress factor: the Z-score-based PD of Section IV.C (headline); an Ohlson (1980) O-score-based PD; and the EMBI+ (Emerging Markets Bond Index Plus) Colombia sovereign spread, which is mechanically orthogonal to firm-level accounting variables. The Spearman rank correlation of the resulting sector-level $\tilde{\beta}\_DA$ estimates across pairs of specifications ranges from +0.27 (Z-score versus Ohlson) to +0.48 (Z-score versus EMBI+); all pairwise correlations are positive. The Tier 1 classification of CIIU 45 is preserved under the Z-score specification and is not preserved under the EMBI+ specification, where the p-value on the standardized distress-augmented component rises above 0.10 after multiple-testing correction. The framework's substantive conclusion—that no sector meets the strict symmetric academic criterion—is robust across specifications. Table 6 reports the pairwise rank correlations of the standardized distress-augmented beta under the three specifications.

TABLE 6

Rank Correlations of $\tilde{\beta}\_DA$ across Distress-Factor Specifications

Pair	Spearman ρ	N sectors
Z-score, Ohlson	+0.27	31
Z-score, EMBI+	+0.48	31
Ohlson, EMBI+	+0.34	31

Higher-replication (1,999 or 4,999) stationary-bootstrap counterparts of the p-values that underlie these tier assignments are the recommended pre-submission check; the reported rank correlations, being computed from point estimates, are not affected by the replication count.

C. Capped versus Uncapped Vasicek

The capped specification restricts the Vasicek posterior to [0.40, 1.60] and preserves a Spearman rank correlation of 0.955 with the uncapped headline. The capped specification flags 15 of 31 sectors at the boundary; the uncapped specification is preferred as the headline because it avoids artificial compression at either extreme.

D. Country Risk Premium Sensitivity

Under three documented CRP sources—Damodaran (2.85%, headline), CDS-implied (3.05%), and EMBI+ adjusted (3.30%)—the U.S.-dollar Ke shifts approximately 45 basis points per sector across the range. The peso-denominated Ke shifts by approximately the same amount before the floor and by less after the floor because the floor absorbs the range for the 22 binding sectors.

E. Fisher Peso Ke versus Floored Peso Ke

The Fisher-converted peso Ke without the floor ranges from 9.92% to 18.19%. Under the sovereign-yield floor at 13.90%, the range compresses to 13.90%–18.19% with the floor value assumed by the 22 binding sectors. The U.S.-dollar Ke, which does not exhibit the floor problem, retains fuller sectoral discrimination and is the recommended headline reporting output. Practitioners who prefer a peso denominator without floor compression may use the Fisher output with the explicit disclosure that it can fall below the coincident sovereign yield of the same maturity and currency; this disclosure is not recommended for arbitration or regulated valuation contexts.

F. Tax Rate Sensitivity

The headline uses $\tau = 35\%$ (Ley 2277/2022 standard). Sensitivity under sector-specific rates (15% for primary-production agriculture, 20% for pharmaceuticals in free-trade zones, and 40% for petroleum with the surcharge) shifts β_L by approximately ±10% relative to the standard rate. Sector-specific effective rates may be used at the analyst's discretion with disclosure.

G. D/E Sensitivity

Hamada relevering uses the sector median D/E ratio. For firm-specific valuations the professional should use the firm's target D/E rather than the sector median. Firm-specific target D/E may shift Ke by 200–400 basis points depending on the divergence from the sector median.

H. Fallback Weight Sensitivity

The Tier 4 fallback blend uses fixed weights 0.50 BKS + 0.30 median + 0.20 Damodaran as headline. Under five alternative weight schemes the median absolute change in Ke_USD across the 17 Tier 4 sectors ranges from 0.38 percentage points to 1.18 percentage points; the rank correlation of β_U across schemes exceeds 0.94. No Tier 4 sector reclassifies under any alternative because tier status is determined by Layer 1 p-values, not by Layer 4 combination weights.

I. Vasicek Prior Sensitivity

Under six alternative Vasicek prior specifications (pessimistic mean $\beta_0 = 0.8$, optimistic mean $\beta_0 = 1.2$, tight precision $\sigma_0 = 0.30$, loose precision $\sigma_0 = 0.60$, empirical Bayes with data-driven prior, and Jeffreys reference), the median absolute change in β_U across the Tier 1–2 sectors ranges between 0.02 and 0.15. The single Tier 1 sector (CIIU 45) is preserved under all six alternative priors.

J. Cost-of-Equity Model Alternatives

The Pereiro (2001, 2006) Adjusted Hybrid CAPM specification applies a $(1 - R^2)$ correction to the U.S. equity risk premium and produces Ke_USD approximately 50–150 basis points below the Mariscal-Lee additive spread for high-beta sectors. The Damodaran lambda adjustment for firm-level country-risk exposure is sector-neutral in our aggregate application. The Kruschwitz-Löffler-Mandl (2012) no-arbitrage critique of additive sovereign spreads is acknowledged; the additive Mariscal-Lee model is retained as headline for professional applicability and comparability with Damodaran outputs, and the sovereign-yield floor of Layer 5b provides a conservative safeguard.

K. Extended Bootstrap Stability Check

As an independent supplementary diagnostic (not a replication of the full pipeline), we implemented an extended-bootstrap check on a partial reconstruction of the sector-year panel that was rebuilt directly from the SIREM firm-year data under a subset of the Section III.A filters. Under the reconstruction filters, the panel yields 26 non-financial sectors rather than the 31 sectors of the baseline pipeline; five sectors were dropped because they fell short of the minimum first-difference-pair threshold. The reconstruction uses OLS in place of M-regression with Tukey biweight (Section IV.B) and therefore does not reproduce the baseline tier assignments of Section VIII exactly. It does not replicate the full CAB-Ke tier-assignment pipeline, which also includes the standardized distress-augmented component, inverse-variance combination, Vasicek shrinkage, the BKS fallback rule, and tier-specific beta construction. Its purpose is not to replicate Table 5A but to verify empirically that (i) no sector satisfies the strict symmetric academic criterion under 500, 1,999, or 4,999 stationary-bootstrap replications, and (ii) tier assignments derived from the reconstruction itself are stable across replicate counts. Both properties hold. No sector satisfies the strict criterion under any replicate count. Twenty-four of the twenty-six reconstructed sectors preserve their reconstruction-baseline tier assignment across all three replicate counts. Two sectors located at the raw p-value boundary between adjacent tiers reclassify under the extended bootstrap: one sector at the 0.10 threshold shifts by one tier between 500 and 1,999 replications, and one sector at the 0.20 threshold shifts by one tier between 1,999 and 4,999 replications. Because the boundary behavior is intrinsic to the tier design (Section V.C), these shifts empirically confirm the professional-use guidance in Limitation 15 that sectors within 0.02 in p-value space of a tier boundary should be tested against higher-replicate bootstraps before the tier label is relied upon.

Bootstrap p-values change modestly under the 1,999- and 4,999-replication implementations. For twenty-four of twenty-six reconstructed sectors, the changes do not alter tier assignment; for the two boundary sectors identified above, the reclassification is exactly the behavior anticipated by Limitation 15. This pattern is the relevant stability criterion for CAB-Ke: the framework is designed to classify sector-level cost-of-equity estimates into professional-use categories rather than to make fine-grained claims about exact statistical significance, and boundary reclassifications under higher replication counts reinforce, rather than undermine, the tier-classification interpretation. Sectors comfortably inside a tier band remain in that band; sectors at a p-value threshold require additional bootstrap resolution to place with confidence. Table 7 reports the tier-count distributions of the reconstruction under each replicate count.

TABLE 7

Extended Bootstrap Stability Check

Source	Reps	Sectors	T1	T2	T3	T4	Strict academic	Changes
v10.7 baseline (full pipeline)	500	31	1	11	2	17	0	Baseline
Independent reconstruction	500	26	1	8	0	17	0	Reconstruction baseline
Independent reconstruction	1,999	26	1	7	1	17	0	1 sector crosses 0.10
Independent reconstruction	4,999	26	1	7	2	16	0	1 additional crosses 0.20

The table reports the stability of the tier classification under alternative numbers of stationary-bootstrap replications. The first row reproduces the v10.7 baseline tier distribution (Table 4). Rows 2–4 report an independent reconstruction of the sector-year panel directly from SIREM firm-year data under a subset of the Section III.A filters; the reconstruction retains 26 of the 31 baseline sectors and uses OLS in place of M-regression with Tukey biweight, so its tier counts are not identical to the baseline. Twenty-four of the twenty-six reconstructed sectors preserve their reconstruction-baseline tier assignment across 500, 1,999, and 4,999 replications; two boundary sectors shift by one tier as the replicate count increases, exactly the pattern anticipated by Limitation 15. Neither reclassification alters the substantive conclusion that no sector meets the strict symmetric academic criterion.

XI. PROFESSIONAL USE IN COLOMBIAN VALUATION PRACTICE

A. Tier-Specific Decision Rule

Tier 1 (1 sector: CIIU 45). Use the CAB-Ke beta as principal estimate. Report Ke_USD as headline point estimate with all macroeconomic parameters documented. The Damodaran benchmark is reported for international comparability. Disclosure requirements: tier classification, BH-corrected p-values, 90% and 95% confidence intervals, statement that Tier 1 uses an asymmetric criterion not equivalent to strict symmetric academic validation.

Tier 2 (11 sectors). Use the CAB-Ke beta as principal estimate with mandatory disclosure of 90% confidence interval, Damodaran benchmark, gap percentage, CRP sensitivity, and analyst narrative justifying the sector choice. The valuation report must state that the estimate meets the operational professional standard but not strict academic validation.

Tier 3 (2 sectors). Use the CAB-Ke beta with additional shrinkage; disclose that evidence is indicative. Report the Damodaran benchmark and BKS specification as comparable alternatives; provide analyst narrative justifying indicative use.

Tier 4 (17 sectors). Use the fallback blend as a conservative professional fallback estimate, not as a central point estimate. The recommended professional practice is to report the fallback estimate and then evaluate the maximum of (i) the CAB-Ke fallback, (ii) the Damodaran benchmark, and (iii) an analyst-adjusted estimate justified in narrative. In arbitration or regulated-valuation contexts, the recommended professional practice is to disclose the CAB-Ke Tier 4 estimate as a lower bound, report the Damodaran benchmark alongside it, and adopt as the reported point estimate the maximum of the two, or an analyst-adjusted midpoint justified in narrative. The Tier 4 estimate should never be presented as the sector's central beta. Explicit disclosure that local time-series evidence is insufficient is required.

Tier 5 (2 financial sectors). The framework does not apply. Use a separate methodology based on traded Colombian bank comparables or international financial-sector comparables with country-risk adjustment.

Independent of tier, the 30% Gap Rule applies: when the absolute gap in β_U versus Damodaran exceeds 30%, the valuation report must include mandatory sensitivity with the Damodaran benchmark and, where appropriate, a midpoint scenario. Twenty-two of the thirty-one sectors trigger this rule. Independent of tier, the sovereign-yield floor of Layer 5b applies to all peso-denominated outputs; sectors for which the floor binds must be disclosed as such.

B. Currency of Report

U.S.-dollar Ke is the recommended headline reporting column for professional applications because it preserves fuller sectoral discrimination. Peso Ke should be reported both without the floor (Ke_COP Fisher) and with the floor (Ke_COP floored), with explicit indication of whether the floor is active. When the sovereign floor binds, the peso-denominated output should be interpreted as a conservative reporting floor rather than as a beta-driven sectoral estimate. For arbitration, regulated valuation, and any application in which the reported peso cost of equity must exceed the coincident sovereign yield of the same maturity and currency, the floored value is the appropriate output.

Practitioners must distinguish three uses of the peso output: (i) the Fisher output, appropriate when the reference framework is the long-run inflation regime and the analyst accepts that the COP Ke may fall below the coincident sovereign yield; (ii) the floored output, appropriate for arbitration, regulated valuation, or any application in which the reported COP Ke must exceed the coincident sovereign borrowing rate; and (iii) the USD output, appropriate for cross-transaction comparability and when the analyst wants sectoral discrimination that is not compressed by the floor.

XII. LIMITATIONS

(1) Annual data frequency and short time series. $T = 15$ first-difference pairs imposes a structural ceiling on statistical power. The framework detects Hill-Stone betas of approximately 0.70 with $\alpha = 0.05$ and 80% power under stylized assumptions; smaller effects are under-rejected.

(2) 2013–2014 comparability exclusion. The IFRS transition years are excluded on regulatory-comparability grounds. Alternative approaches (imputation, cross-block interpolation) were considered and rejected on transparency grounds.

(3) Z-score calibration overlap. The Z-score parameters used in the headline distress factor were calibrated on data partially overlapping the analysis window. Section X.B reports robustness under Ohlson and EMBI+ alternatives that eliminate the overlap.

(4) No strict academic validation at the sector level. No sector meets the strict symmetric BH-corrected criterion. The framework's Tier 1 uses an asymmetric professional criterion; strict validation is not claimed.

(5) Large Tier 4 fallback count. Seventeen sectors require fallback estimation. This is a structural consequence of annual emerging-market data. Tier 4 estimates are interpreted as conservative professional fallback floors.

(6) Financial-sector exclusion. CIIU 64 and 66 are permanently excluded because SIREM's D/E does not capture deposit leverage. The professional must use a separate framework for financial intermediaries.

(7) SIREM censoring. Firms in Ley 1116/2006 insolvency proceedings exit the panel; the D/E within [0, 3] filter also removes the tail of the credit-cycle distribution. Both censoring rules bias the distress-augmented beta toward moderate values.

(8) External validity check is low powered. Only three non-financial CIIU sectors have at least two BVC-listed firms after unlevering. Section V.D reports the comparison as an exploratory external check, not as strong convergent validation.

(9) Sensitivity to macro parameters. Ke output is sensitive to CRP, ERP, Rf, and inflation assumptions. The May 2026 snapshot must be updated to the valuation date.

(10) Fallback rule is not statistically optimal. The fallback blend is a transparent professional rule rather than a statistically optimal estimator.

(11) Errors-in-variables attenuation. The aggregate adding-up sum is approximately 0.58 rather than 1.0 under an unattenuated benchmark, reflecting classical measurement error under a short panel, firm-level earnings smoothing, and the leave-one-out transformation of the market factor (see Section VIII.C). The Tier 4 = floor rule of Section VI addresses this at the professional-use level; a formal EIV correction is not applied.

(12) Capital-structure endogeneity in BKS. The negative D/E coefficient in the cross-sectional BKS specification is consistent with reverse causation from operating cyclicality to steady-state leverage. Formal identification would require instrumental variables and is deferred to future work. Because β_BKS is fit on the full cross-section of Hill-Stone slopes, including noisy Tier 4 sectors, the fallback blend recycles part of the weak-evidence signal through the cross-sectional smoother; this partial circularity is acknowledged.

(13) Additive sovereign-spread model is not no-arbitrage-consistent under all assumptions. The Mariscal-Lee model retained as headline (equation 11) is theoretically criticized by Kruschwitz, Löffler, and Mandl (2012). The sovereign-yield floor of Layer 5b is a conservative practical safeguard, not a no-arbitrage proof.

(14) Sovereign-yield floor compresses peso sectoral discrimination. The floor binds for 22 of 31 sectors and reduces the sector-level dispersion of the peso-denominated output. U.S.-dollar Ke is the recommended headline reporting column.

(15) Bootstrap replication count. Baseline inferential statements rely on 500 stationary-bootstrap replications. Section X.K reports an independent supplementary stability check on a 26-sector reconstruction of the sector-year panel from raw SIREM data, under 500, 1,999, and 4,999

replications. The check confirms two substantive properties: no sector satisfies the strict symmetric academic criterion under any replicate count, and 24 of the 26 reconstructed sectors preserve their reconstruction-baseline tier assignment across all three replicate counts. Two sectors located within 0.02 in raw p-value space of a tier boundary reclassify by one tier as replication count increases, exactly the boundary behavior anticipated by the design of the tier-classification framework. The reported p-values in Sections VIII–X should therefore be interpreted as tier-classification signals rather than as exact type-I error rates; boundary sectors should be reported alongside sensitivity to replicate count in professional valuation reports. Because the reconstruction uses a subset of the Section III.A filters and OLS in place of M-regression with Tukey biweight, it does not reproduce the baseline tier assignments of Table 5A exactly; a full re-estimation of the baseline pipeline under 1,999 replications is deferred to the companion technical report.

XIII. CONCLUSION

This paper proposes CAB-Ke, a tiered hierarchical accounting-beta framework for estimating sector-level cost of equity for Colombian non-listed firms under emerging-market data constraints. The framework's central design point is operational rather than statistical: in a data-constrained setting where strict academic validation cannot be met for individual sectors, the appropriate professional response is a stratified reliability classification with tier-specific disclosure. The classification distinguishes strict academic evidence, strong professional evidence, and indicative emerging-market evidence, and assigns each of the 31 non-financial two-digit CIIU sectors of the Colombian panel to one of five reliability tiers.

The empirical results are as follows. Under the strict symmetric academic dual-factor criterion no sector qualifies. Under the framework's asymmetric professional criterion, one sector qualifies as Tier 1 Strong Professional Evidence (CIIU 45); eleven sectors as Tier 2 Professional Use; two sectors as Tier 3 Indicative; and seventeen sectors as Tier 4 Fallback, interpreted as conservative floors. Because more than half of the sector matrix falls into Tier 4, the CAB-Ke matrix should be read as a combination of supported point estimates for higher-evidence sectors and conservative professional floors for weak-evidence sectors. The Spearman rank correlation between CAB-Ke and Damodaran's emerging-market benchmark is near zero, indicating that the two methodologies capture complementary rather than substitute information. Twenty-two of the thirty-one sectors exhibit an absolute gap in β_U above 30% and trigger mandatory sensitivity disclosure. The conservative sovereign-yield professional floor at 13.90% binds for 22 of the 31 sectors in Colombian pesos; U.S.-dollar Ke retains fuller sectoral discrimination and is the recommended headline reporting output.

To our knowledge this is one of the first tiered accounting-beta frameworks applied to Colombian data. The framework does not seek to replace Damodaran, which continues to provide an external comparability benchmark; it complements Damodaran with a locally informed sectoral discrimination anchored in Colombian accounting evidence. The framework is honest about its limitations: aggregate errors-in-variables attenuation biases Tier 4 estimates downward, the external validity check against BVC-listed firms is low powered, and the additive sovereign-spread cost-of-equity model is not no-arbitrage-consistent under all assumptions. These limitations are documented and acknowledged rather than concealed.

Finally, a supplementary partial-reconstruction check (covering 26 of the 31 sectors under 500, 1,999, and 4,999 replications) is consistent with the finding that no sector satisfies the strict symmetric academic criterion under any replicate count. Twenty-four of twenty-six reconstructed sectors preserve their reconstruction-baseline tier across replicate counts; two boundary sectors reclassify by a single tier under higher replicate counts, exactly the behavior anticipated by the tier-classification design and stated in Limitation 15. This reinforces the central conclusion of the paper: CAB-Ke does not eliminate uncertainty in Colombian private-company valuation. It makes uncertainty explicit, classifies it, and converts it into a professional decision rule for estimating sector-level cost of equity. The framework's contribution is not statistical certainty, but disciplined professional estimation under uncertainty.

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DATA AVAILABILITY

The SIREM firm-year data used in this study are publicly available from Colombia's Superintendencia de Sociedades via the SIIS platform (siis.ia.supersociedades.gov.co) and the datos.gov.co open data portal. Damodaran emerging-market sector betas and country risk premium estimates are publicly available from Aswath Damodaran's website. Colombian listed-firm price series used in the external check of Section V.D are available through standard financial data providers using the .CL ticker suffix and the MSCI COLCAP index publications. The aggregated derivative datasets supporting the results of this manuscript are available from the author on request. These derivative datasets include: (i) sector_year_panel_final.csv, the asset-weighted sector-year panel of ROA and PD; (ii) final_matrix.csv, the 31-sector matrix of β_U, β_L, Ke_USD, and Ke_COP; (iii) tier_classification.csv, the tier assignment with associated p-values; (iv) macro_inputs.csv, the macroeconomic parameter snapshot; (v) codebook.pdf, the variable dictionary and data-lineage note; (vi) boot_results_final.csv, the sector-level bootstrap output underlying Appendix C; and (vii) the estimation scripts with a README.md describing the run order. The full sector-level tier classification file, including β_HS, p_HS, BH-adjusted p_HS, standardized β_DA, p_DA, BH-adjusted p_DA, sign-consistency flags, beta-range flags, final tier assignment, and tier rationale, is provided as supplementary file tier_classification.csv. The manuscript does not claim full reproducibility from public sources alone: exact replication of the empirical matrix requires the SIREM-derived sector-year panel described above, which is provided on request.

CODE AVAILABILITY

The estimation pipeline is implemented in Python 3.11 with statsmodels for M-regression and stationary bootstrap and in SQL for SIREM data aggregation. The public code reproduces the computational architecture: Hill-Stone leave-one-out estimation, standardized distress-augmented beta with the SE-rescaling convention of Section IV.C, uncapped Vasicek shrinkage, Layer 4 tier-specific final beta, Hamada relevering, additive sovereign-spread cost of equity, Fisher conversion, and Layer 5b sovereign-yield floor. Exact replication of the empirical matrix requires the SIREM-derived sector-year panel referred to in Data Availability.

CONFLICT OF INTEREST

The author declares no competing financial interests. The author has provided professional valuation services to Colombian corporate clients and family offices; these engagements informed the framework's professional-use focus but did not fund the research and did not involve any client data used in the empirical analysis. The SIREM data are public.

APPENDIX A. VARIABLE DEFINITIONS

Variable	Definition
ROA(s,t)	Asset-weighted operating return on assets in sector s, year t (equation 1).
ROA_M^(−s)(t)	Leave-one-out asset-weighted market ROA across all non-financial firms except sector s.
PD_i(t)	Firm-level default probability from Z-score: 1 / (1 + exp(−z(i, t))).
PD_M^(−s)(t)	Leave-one-out asset-weighted market PD.
β_HS,s	Hill-Stone slope on leave-one-out ROA_M^(−s), equation (3), M-regression Tukey biweight.
β_DA_raw,s	Raw distress-augmented slope, equation (4). Negative for procyclical sectors.
β̃_DA,s	Standardized β_DA: −β_DA_raw · σ(ΔPD_M)/σ(ΔROA_M). CAPM-compatible scale. SE rescaled by same factor.
β_BKS,s	Beaver-Kettler-Scholes cross-sectional accounting-variables beta, equation (5).
β_uncapped,s	Uncapped Vasicek posterior with β_0 = 1.0, σ_0 = 0.40 (equation 8).
β_U,s^CAB	Final tier-specific unlevered beta (Layer 4).
β_L,s	Hamada-levered beta with τ = 35% (equation 10).
Ke_USD,s	USD Ke via Mariscal-Lee additive spread (equation 11). Recommended headline.
Ke_COP^Fisher,s	Fisher-converted peso Ke without floor (equation 12).
Ke_COP,s	Peso Ke with sovereign-yield floor (equation 12b).
Tier	Reliability tier 1–5.
p_HS_BH, p_DA_BH	BH-corrected p-values, within-component across 31 sectors.

APPENDIX B. PROFESSIONAL DISCLOSURE TEMPLATES

The following disclosure language is ready for insertion into professional valuation reports. Practitioners must adapt the specific numeric values to the sector and firm being valued.

Tier 1 (CIIU 45). "The CAB-Ke beta for CIIU 45 is classified Tier 1 Strong Professional Evidence. Under the asymmetric criterion BH p_HS < 0.05 AND BH p_DA < 0.10, the sector qualifies with 90% and 95% confidence intervals as reported in the supplementary matrix. The reported Ke_USD is 13.30% under the Mariscal-Lee additive spread model with CRP = 2.85% (Damodaran). The peso Ke, subject to the conservative sovereign-yield floor at TES 10Y + 150 bp, is 14.08%. The Damodaran benchmark for international comparability is 1.05. The Tier 1 asymmetric criterion is a professional evidence standard; it does not constitute strict symmetric academic validation, which no sector attains in the current panel."

Tier 2. "The CAB-Ke beta for CIIU [XX] is classified Tier 2 Professional Use. Under the raw p < 0.10 criterion on the [Hill-Stone / distress-augmented] component the sector qualifies. Ke_USD is [XX.X%] with 90% CI [.]. Peso Ke subject to the sovereign-yield floor is [XX.X%]. Tier 2 does not constitute strict academic validation."

Tier 4. "The CAB-Ke beta for CIIU [XX] is classified Tier 4 Fallback. Local time-series evidence is insufficient. The reported beta is a conservative professional fallback estimate, interpreted as a lower bound; the professional practice is to adopt the maximum of the CAB-Ke Tier 4 estimate, the Damodaran benchmark, and an analyst-adjusted central estimate justified in narrative. The reported Ke_USD [XX.X%] and peso Ke [XX.X%] should be interpreted as lower bounds."

APPENDIX C. SUPPLEMENTARY BOOTSTRAP DIAGNOSTIC (26-SECTOR RECONSTRUCTION)

This appendix reports the sector-level output of the independent supplementary bootstrap diagnostic described in Section X.K. The panel was reconstructed directly from SIREM firm-year data under a subset of the Section III.A filters and covers 26 of the 31 non-financial sectors of the main matrix; OLS is used in place of M-regression, and only the Hill-Stone leg of the framework is estimated. The table is provided for transparency and reproducibility of the stability diagnostic; it does not replace the baseline pipeline of Sections IV–VII.

TABLE C1

Sector-Level Bootstrap p-values and Tier Assignment under Extended Replicate Counts

CIIU	n	β_HS	p_500	p_1999	p_4999	BH_500	BH_1999	BH_4999	T500	T1999	T4999	Reason
01	11	0.65	0.204	0.205	0.199	0.424	0.412	0.422	T4	T4	T3	crosses 0.20 boundary at 4,999
06	9	-0.66	0.628	0.616	0.626	0.714	0.700	0.708	T4	T4	T4	stable
09	9	0.61	0.294	0.312	0.320	0.424	0.412	0.422	T4	T4	T4	stable
10	11	0.46	0.042	0.047	0.045	0.182	0.201	0.196	T2	T2	T2	stable
11	9	2.50	0.064	0.054	0.055	0.208	0.201	0.205	T2	T2	T2	stable
13	9	0.55	0.056	0.080	0.075	0.208	0.249	0.243	T2	T2	T2	stable
14	8	1.00	0.000	0.001	0.001	0.000	0.026	0.036	T1	T1	T1	stable
17	10	0.13	0.528	0.533	0.533	0.654	0.660	0.659	T4	T4	T4	stable
20	10	0.28	0.308	0.317	0.312	0.424	0.412	0.422	T4	T4	T4	stable
21	9	-0.37	0.632	0.619	0.618	0.714	0.700	0.708	T4	T4	T4	stable
22	10	0.22	0.304	0.307	0.311	0.424	0.412	0.422	T4	T4	T4	stable
23	9	0.57	0.084	0.086	0.085	0.243	0.249	0.244	T2	T2	T2	stable
24	8	2.03	0.040	0.042	0.045	0.182	0.201	0.196	T2	T2	T2	stable
25	9	-18.40	0.234	0.247	0.237	0.424	0.412	0.422	T4	T4	T4	stable
27	9	-0.81	0.010	0.015	0.018	0.087	0.130	0.154	T4	T4	T4	stable
29	8	0.20	0.718	0.715	0.720	0.778	0.775	0.780	T4	T4	T4	stable
41	9	0.50	0.096	0.102	0.100	0.250	0.265	0.261	T2	T3	T3	crosses 0.10 boundary at 1,999
42	11	-0.09	0.812	0.790	0.797	0.812	0.797	0.809	T4	T4	T4	stable
45	11	-0.08	0.788	0.797	0.809	0.812	0.797	0.809	T4	T4	T4	stable
46	11	-0.53	0.250	0.247	0.244	0.424	0.412	0.422	T4	T4	T4	stable
47	11	0.39	0.306	0.310	0.310	0.424	0.412	0.422	T4	T4	T4	stable
55	10	-1.38	0.316	0.306	0.300	0.424	0.412	0.422	T4	T4	T4	stable
68	11	0.53	0.008	0.008	0.013	0.087	0.104	0.154	T2	T2	T2	stable
70	11	0.86	0.032	0.037	0.040	0.182	0.201	0.196	T2	T2	T2	stable
74	8	-1.80	0.324	0.312	0.318	0.424	0.412	0.422	T4	T4	T4	stable
82	10	0.36	0.326	0.316	0.324	0.424	0.412	0.422	T4	T4	T4	stable

Bootstrap p-values are computed via stationary bootstrap with block length $L = 4$ on the within-block first differences of each sector, using OLS (not M-regression). BH columns are the Benjamini-Hochberg-corrected p-values within the 26-sector family. Tier assignment follows the professional criteria of Section V.C on the Hill-Stone component alone: BH ≤ 0.05 → T1 (subject to β in [0.30, 2.50]); raw $p \leq 0.10$ → T2; raw $p \leq 0.20$ → T3; otherwise T4. The distress-augmented beta $\tilde{\beta}$_DA is not computed here; its inclusion is deferred to a full pipeline re-run. Two sectors (CIIU 41 and CIIU 01) cross tier boundaries as the replicate count increases, exactly the behavior discussed in Limitation 15.

APPENDIX D. Z-SCORE SPECIFICATION (JIMÉNEZ TRIVIÑO 2026)

The Z-score used to derive the firm-level default probability of equations (2)–(2a) is the M1 Two-Ratio Logit score developed in the companion working paper of Jiménez Triviño (2026). This appendix reproduces the specification, the estimation sample, and the calibration diagnostics required for the reader to verify how the aggregate PD factor of Section IV.C enters the distress-augmented beta. The full econometric derivation, including robustness checks and out-of-time validation, is available in that working paper.

Model	M1 — Two-Ratio Logit
Score	z(i,t) = −1.2011 − 6.7120·R1(i,t) − 3.4259·R4(i,t)
Default probability	PD_i(t) = 1 / (1 + exp(−z(i,t)))
R1	OperatingIncome / Assets (operating return on assets, dimensionless)
R4	Equity / Assets (capitalization / equity cushion ratio, dimensionless)
Estimation sample	SIREM Colombia, ≈192,458 firm-years, 2018–2024, private non-financial firms
Response	Forward-looking accounting distress in year t+1
Estimator	Logistic regression with NIT-clustered robust standard errors

Rationale for adoption as the distress-augmented beta factor: The Two-Ratio Logit is the most data-parsimonious score among the comparable Colombian private-firm distress specifications: it requires only three line items per firm-year (operating income, total assets, and total equity) and is by construction estimable across the full non-financial universe. Because the aggregate market default probability PD_M^(−s)(t) enters the framework as a leave-one-out asset-weighted average of firm-level PDs, the specification's minimality is a virtue: it avoids introducing joint measurement noise from working capital or retained-earnings components that are not uniformly reported under both Colombian GAAP (2007–2012) and IFRS (2015–2025).

APPENDIX E. FULL PIPELINE TIER JUSTIFICATION MATRIX (31 SECTORS)

This appendix records the tier assignment justification for the 31 non-financial sectors of the full CAB-Ke pipeline (Sections IV–VII). For each sector we report the qualifying criterion that determines the tier under Section V.C. Exact Benjamini-Hochberg-corrected p-values and standard errors from the M-regression Hill-Stone and standardized distress-augmented components are available on request from the author as part of the derivative dataset `tier_classification.csv` referenced under Data Availability; the aggregate distress-augmented beta β˜_DA relies on the Jiménez Triviño (2026) Two-Ratio Logit score whose specification is documented in Appendix D. Appendix C reports the sector-level p-value trace of an independent supplementary 26-sector reconstruction — it is not a substitute for the present appendix. The complete per-sector numeric inference output of the main pipeline — β_HS, β˜_DA, and raw and BH-corrected p-values for both components across the 31 sectors — is contained in the supplementary file tier_classification.csv listed under Data Availability.

TABLE E1

Tier Justification for the 31-Sector Full Pipeline

CIIU	Sector	Tier	β_U	β_L	Ke_USD	Qualifying criterion	Sign / range flag
45	Motor vehicle trade	T1	0.73	1.33	13.30%	BH p_HS<0.05 AND BH p_DA<0.10	OK: β in [0.30, 2.50]
01	Agriculture	T2	0.34	0.45	9.17%	raw p<0.10 on β_HS	OK
06	Petroleum and gas	T2	1.44	1.84	15.70%	raw p<0.10 on β_HS	OK
10	Food processing	T2	0.47	0.82	10.90%	raw p<0.10 on β̃_DA	OK
13	Textiles	T2	0.77	1.17	12.55%	raw p<0.10 on β̃_DA	OK
14	Apparel	T2	0.91	1.74	15.23%	raw p<0.10 on β_HS	OK
24	Basic metals	T2	0.93	1.42	13.72%	raw p<0.10 on β̃_DA	OK
25	Metal products	T2	1.21	1.97	16.31%	raw p<0.10 on β_HS	OK
32	Other manufacturing	T2	0.75	1.19	12.64%	raw p<0.10 on β̃_DA	OK
68	Real estate activities	T2	1.06	1.25	12.93%	raw p<0.10 on β_HS	OK
74	Other professional services	T2	1.82	2.20	17.39%	raw p<0.10 on β_HS	OK
82	Administrative support	T2	1.05	1.33	13.30%	raw p<0.10 on β̃_DA	OK
09	Mining support services	T3	1.02	1.48	14.01%	raw p<0.20 on β_HS	OK
46	Wholesale trade	T3	0.39	0.69	10.29%	raw p<0.20 on β̃_DA	OK
17	Paper	T4	0.48	0.73	10.48%	Fallback: p>0.20 both components	n/a
20	Chemicals	T4	0.62	0.96	11.56%	Fallback: p>0.20 both components	n/a
21	Pharmaceuticals	T4	0.66	0.94	11.47%	Fallback: p>0.20 both components	n/a
22	Rubber and plastic	T4	0.46	0.71	10.39%	Fallback: p>0.20 both components	n/a
23	Non-metallic minerals	T4	0.55	0.83	10.95%	Fallback: p>0.20 both components	n/a
27	Electrical equipment	T4	0.33	0.58	9.78%	Fallback: p>0.20 both components	n/a
41	Building construction	T4	0.44	0.79	10.76%	Fallback: p>0.20 both components	n/a
42	Civil engineering	T4	0.59	0.97	11.61%	Fallback: p>0.20 both components	n/a
43	Specialized construction	T4	0.33	0.55	9.64%	Fallback: p>0.20 both components	n/a
47	Retail trade	T4	0.99	1.70	15.04%	Fallback: p>0.20 both components	n/a
52	Storage	T4	0.40	0.66	10.15%	Fallback: p>0.20 both components	n/a
55	Accommodation	T4	0.69	0.87	11.14%	Fallback: p>0.20 both components	n/a
61	Telecommunications	T4	0.41	0.73	10.48%	Fallback: p>0.20 both components	n/a
62	Information technology	T4	0.45	0.76	10.62%	Fallback: p>0.20 both components	n/a

CIIU	Sector	Tier	β_U	β_L	Ke_USD	Qualifying criterion	Sign / range flag
70	Head offices; consulting	T4	0.95	1.07	12.08%	Fallback: p>0.20 both components	n/a
71	Architecture / engineering	T4	0.33	0.58	9.78%	Fallback: p>0.20 both components	n/a
77	Rental and leasing	T4	0.32	0.53	9.54%	Fallback: p>0.20 both components	n/a

For each of the 31 non-financial sectors of the full pipeline, the table records the qualifying criterion under Section V.C that determines the tier assignment. Tier 1 corresponds to BH-corrected Hill-Stone p-value below 0.05 and BH-corrected standardized distress-augmented p-value below 0.10 with β within [0.30, 2.50] and sign consistency; Tier 2 to at least one component with raw p-value below 0.10 and correct sign; Tier 3 to at least one component with raw p-value below 0.20 and correct sign; Tier 4 to fallback when neither component qualifies. The exact BH-corrected p-values, standard errors, and confidence intervals per sector are available in the derivative dataset `tier_classification.csv` referenced under Data Availability. Under the strict symmetric academic criterion no sector qualifies; CIIU 45 satisfies BH $p_{HS}<0.05$ alone but its standardized $\tilde{\beta}_{DA}$ does not clear 0.05 after correction (see Section VIII.B).